

# CS 4495 Computer Vision

## *Features 1 – Harris and other corners*

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Corners in A



Corners in B

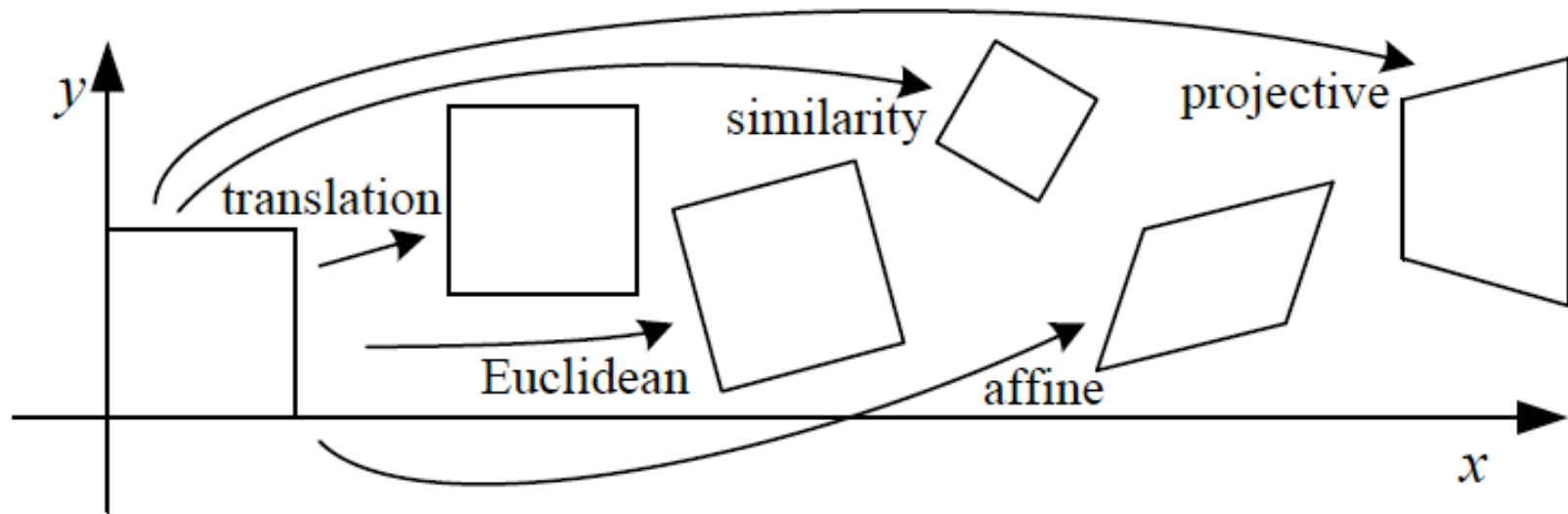


# Administrivia

- PS 4: Out and not changed. Due Sunday Oct 12<sup>th</sup>, 11:55pm
  - It is application of the last few lectures. Mostly straight forward Matlab but if you're linear algebra is rusty it can take a while to figure out. Question 2 takes some doing to understand.
    - You have been warned...
  - It is cool
    - You have been warned...
- Today: Start on *features*.
  - Forsyth and Ponce: 5.3-5.4
  - Szeliski also covers this well – Section 4 – 4.1.1
  - These next 3 lectures will provide detail for Project 4.

# The basic image point matching problem

- Suppose I have two images related by some transformation. Or have two images of the same object in different positions.
- How to find the transformation of image 1 that would align it with image 2?



# We want *Local*<sub>(1)</sub> *Features*<sub>(2)</sub>

- Goal: Find points in an image that can be:
  - Found in other images
  - Found precisely – well localized
  - Found reliably – well matched
- Why?
  - Want to compute a fundamental matrix to recover geometry
  - Robotics/Vision: See how a bunch of points move from one frame to another. Allows computation of how camera moved -> depth -> moving objects
  - Build a panorama...

# Suppose you want to build a panorama



# How do we build panorama?

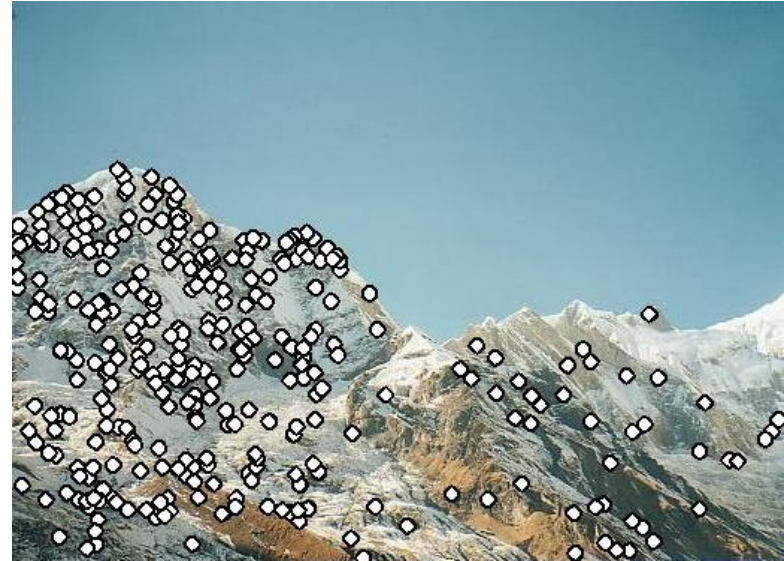
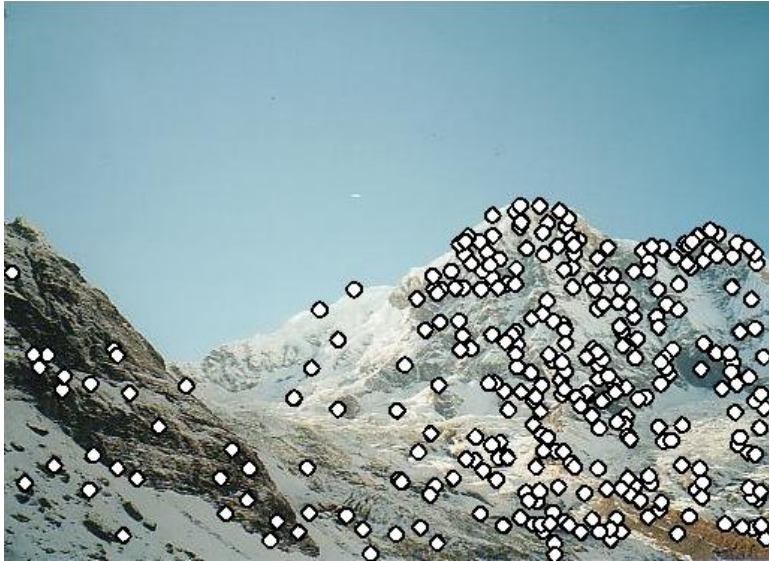
- We need to match (align) images





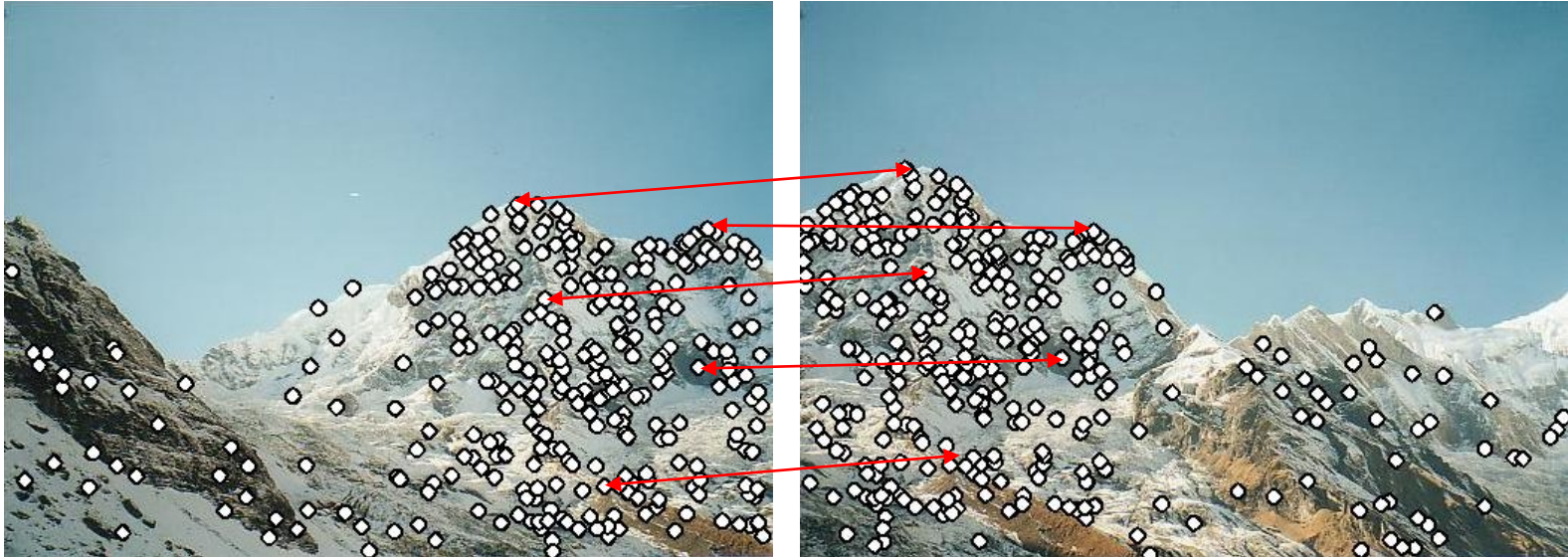
# Matching with Features

- Detect features (feature points) in both images



# Matching with Features

- Detect features (feature points) in both images
- Match features - find corresponding pairs





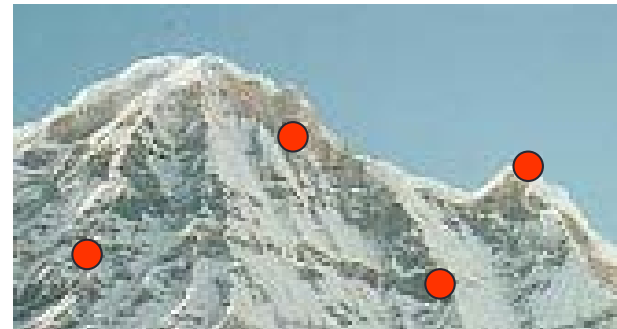
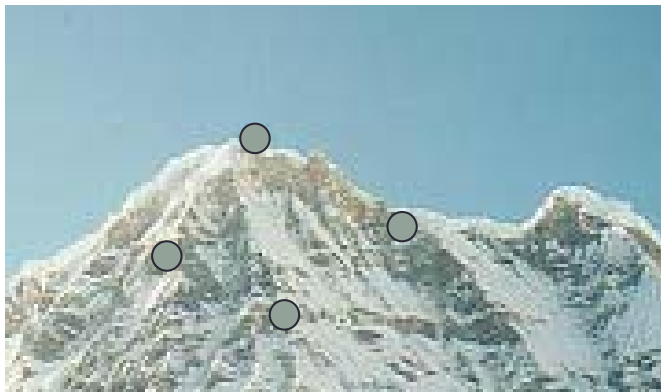
# Matching with Features

- Detect features (feature points) in both images
- Match features - find corresponding pairs
- Use these pairs to align images



# Matching with Features

- Problem 1:
  - Detect the *same* point *independently* in both images



no chance to match!

We need a repeatable detector

# Matching with Features

- Problem 2:
  - For each point correctly recognize the corresponding one

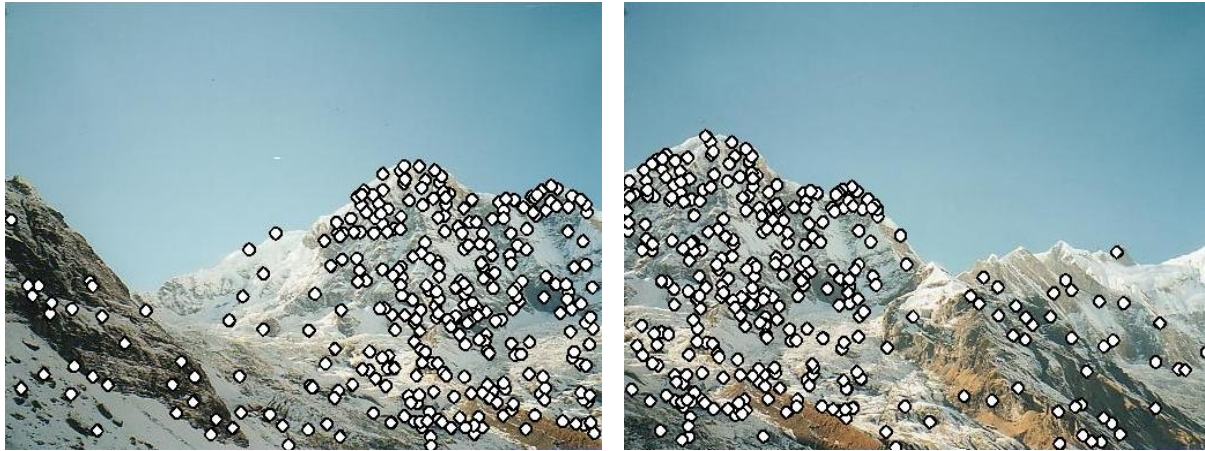


We need a reliable and distinctive *descriptor*

# More motivation...

- Feature points are used also for:
  - Image alignment (e.g. homography or *fundamental* matrix)
  - 3D reconstruction
  - Motion tracking
  - Object recognition
  - Indexing and database retrieval
  - Robot navigation
  - ... other

# Characteristics of good features



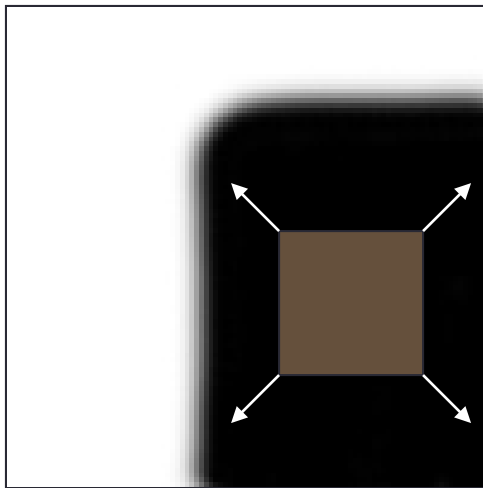
- **Repeatability/Precision**
  - The same feature can be found in several images despite geometric and photometric transformations
- **Saliency/Matchability**
  - Each feature has a distinctive description
- **Compactness and efficiency**
  - Many fewer features than image pixels
- **Locality**
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion



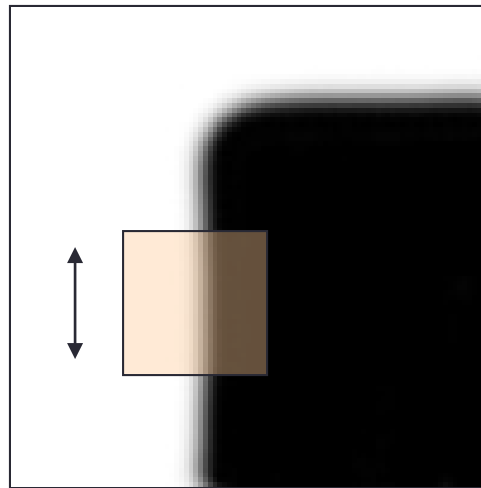


# Corner Detection: Basic Idea

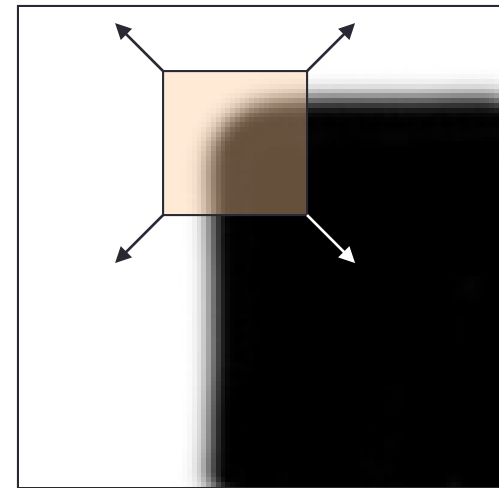
- We should easily recognize the point by looking through a small window
- Shifting a window in any direction should give a large change in intensity



“flat” region:  
no change in  
all directions

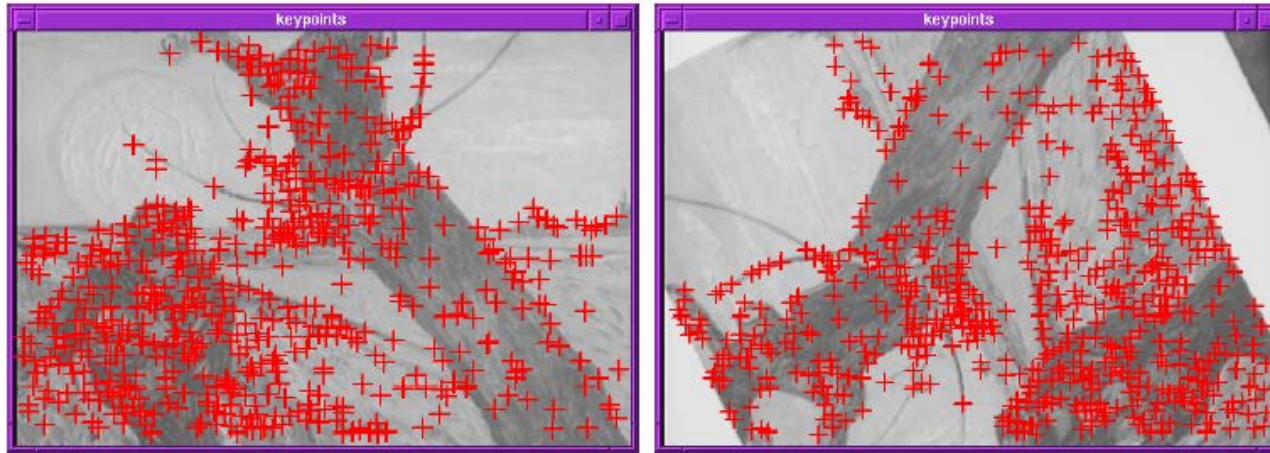


“edge”:  
no change  
along the edge  
direction



“corner”:  
significant change  
in all directions with  
small shift

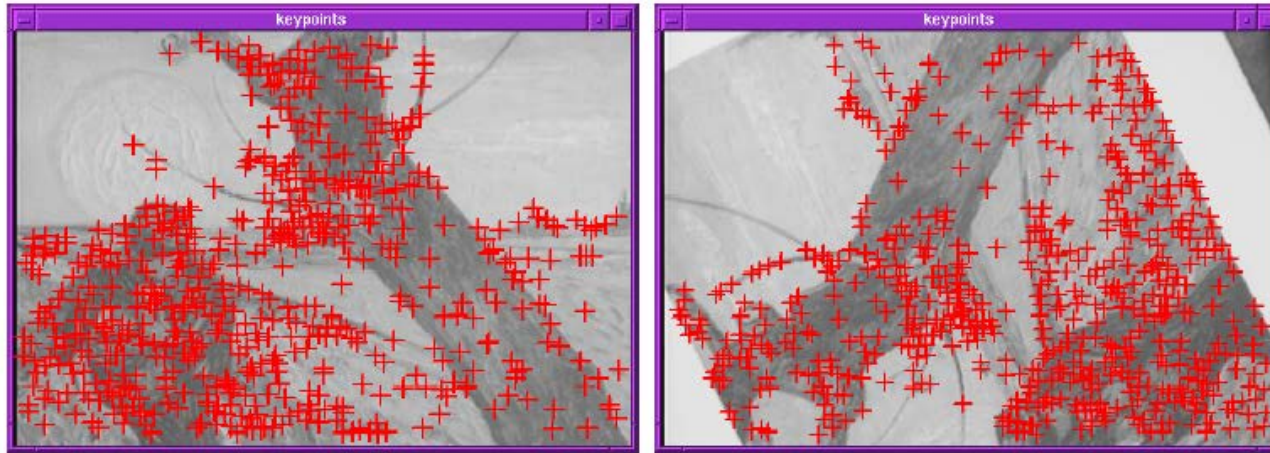
# Finding *Corners*



- Key property: in the region around a corner, image gradient has two or more dominant directions
- Corners are repeatable and distinctive

C.Harris and M.Stephens. ["A Combined Corner and Edge Detector."](#)  
*Proceedings of the 4th Alvey Vision Conference*: pages 147—151, **1988**

# Finding Harris Corners



- Key property: in the region around a corner, image gradient has two or more dominant directions
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C. **Harris** and M. Stephens. ["A Combined Corner and Edge Detector."](#)  
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# Corner Detection: Mathematics

Change in appearance for the shift  $[u, v]$ :

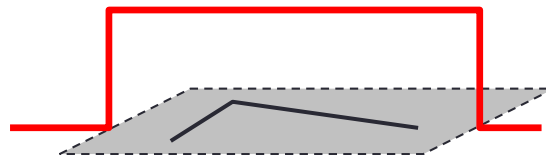
$$E(u, v) = \sum_{x, y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

Window  
function

Shifted  
intensity

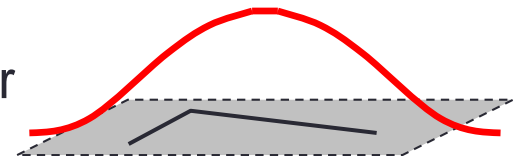
Intensity

Window function  $w(x, y) =$



1 in window, 0 outside

or



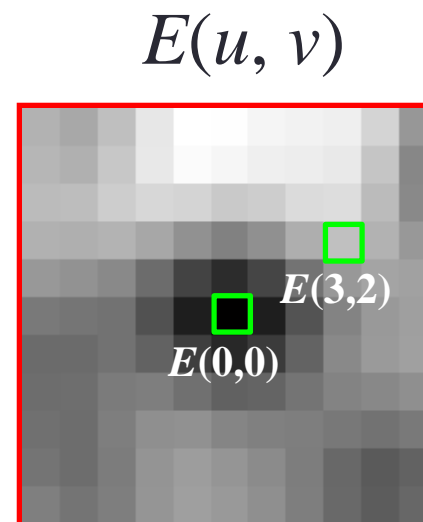
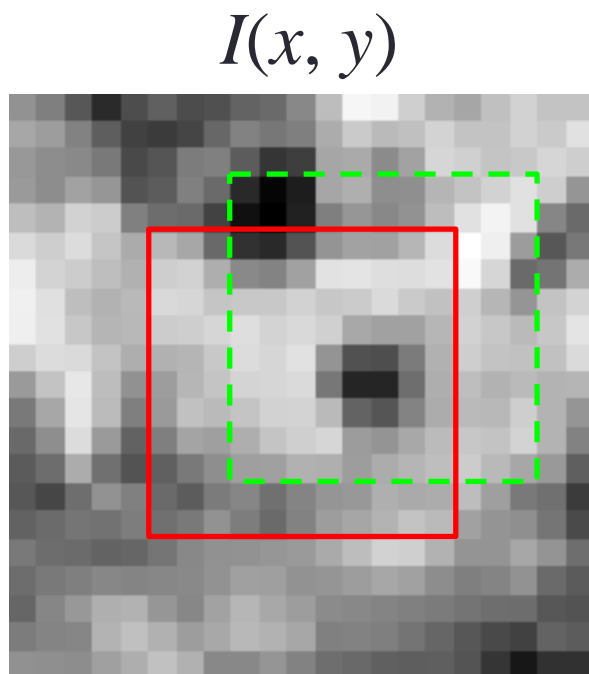
Gaussian



# Corner Detection: Mathematics

Change in appearance for the shift  $[u, v]$ :

$$E(u, v) = \sum_{x, y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$



# Corner Detection: Mathematics

Change in appearance for the shift  $[u, v]$ :

$$E(u, v) = \sum_{x, y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

We want to find out how this function behaves for ***small*** shifts ( $u, v$  near 0,0)

# Corner Detection: Mathematics

Change in appearance for the shift  $[u, v]$ :

$$E(u, v) = \sum_{x, y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

Second-order Taylor expansion of  $E(u, v)$  about  $(0, 0)$   
(local quadratic approximation for small  $u, v$ ):

$$(1D): F(\delta x) \approx F(0) + \delta x \cdot \frac{dF(0)}{dx} + \frac{1}{2} \delta x^2 \cdot \frac{d^2 F(0)}{dx^2}$$

$$E(u, v) \approx E(0, 0) + [u \quad v] \begin{bmatrix} E_u(0, 0) \\ E_v(0, 0) \end{bmatrix} + \frac{1}{2} [u \quad v] \begin{bmatrix} E_{uu}(0, 0) & E_{uv}(0, 0) \\ E_{uv}(0, 0) & E_{vv}(0, 0) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

# Corner Detection: Mathematics

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$$E_u(u, v) = \sum_{x, y} 2w(x, y) [I(x + u, y + v) - I(x, y)] I_x(x + u, y + v)$$

$$E_{uu}(u, v) = \sum_{x, y} 2w(x, y) I_x(x + u, y + v) I_x(x + u, y + v) \\ + \sum_{x, y} 2w(x, y) [I(x + u, y + v) - I(x, y)] I_{xx}(x + u, y + v)$$

$$E_{uv}(u, v) = \sum_{x, y} 2w(x, y) I_y(x + u, y + v) I_x(x + u, y + v) \\ + \sum_{x, y} 2w(x, y) [I(x + u, y + v) - I(x, y)] I_{xy}(x + u, y + v)$$

# Corner Detection: Mathematics

$$E(u, v) = \sum_{x, y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

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# Corner Detection: Mathematics

$$E(u, v) = \sum_{x, y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

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$$\begin{aligned} E_{uu}(u, v) &= \sum_{x, y} 2w(x, y) I_x(x + u, y + v) I_x(x + u, y + v) \\ &\quad + \sum_{x, y} 2w(x, y) [I(x + u, y + v) - I(x, y)] I_{xx}(x + u, y + v) \end{aligned}$$

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# Corner Detection: Mathematics

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# Corner Detection: Mathematics

$$E(u, v) = \sum_{x, y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

Evaluate at  $(u, v) = (0, 0)$ :

$$E(u, v) \approx \boxed{E(0, 0)} + [u \ v] \begin{bmatrix} E_u(0, 0) \\ E_v(0, 0) \end{bmatrix} + \frac{1}{2} [u \ v] \begin{bmatrix} E_{uu}(0, 0) & E_{uv}(0, 0) \\ E_{uv}(0, 0) & E_{vv}(0, 0) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$E_u(u, v) = \sum_{x, y} 2w(x, y) \boxed{[I(x + u, y + v) - I(x, y)]} I_x(x + u, y + v) = 0$$

$$E_{uu}(u, v) = \sum_{x, y} 2w(x, y) I_x(x + u, y + v) I_x(x + u, y + v) + \sum_{x, y} 2w(x, y) \boxed{[I(x + u, y + v) - I(x, y)]} I_{xx}(x + u, y + v) = 0$$

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# Corner Detection: Mathematics

$$E(u, v) = \sum_{x, y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

Second-order Taylor expansion of  $E(u, v)$  about  $(0, 0)$ :

$$E(u, v) \approx E(0, 0) + [u \ v] \begin{bmatrix} E_u(0, 0) \\ E_v(0, 0) \end{bmatrix} + \frac{1}{2} [u \ v] \begin{bmatrix} E_{uu}(0, 0) & E_{uv}(0, 0) \\ E_{uv}(0, 0) & E_{vv}(0, 0) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$E(0, 0) = 0$$

$$E_u(0, 0) = 0$$

$$E_v(0, 0) = 0$$

$$E_{uu}(0, 0) = \sum_{x, y} 2w(x, y) I_x(x, y) I_x(x, y)$$

$$E_{vv}(0, 0) = \sum_{x, y} 2w(x, y) I_y(x, y) I_y(x, y)$$

$$E_{uv}(0, 0) = \sum_{x, y} 2w(x, y) I_x(x, y) I_y(x, y)$$

# Corner Detection: Mathematics

$$E(u, v) = \sum_{x, y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

Second-order Taylor expansion of  $E(u, v)$  about  $(0, 0)$ :

$$E(u, v) \approx [u \ v] \begin{bmatrix} \sum_{x, y} w(x, y) I_x^2(x, y) & \sum_{x, y} w(x, y) I_x(x, y) I_y(x, y) \\ \sum_{x, y} w(x, y) I_x(x, y) I_y(x, y) & \sum_{x, y} w(x, y) I_y^2(x, y) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$E(0, 0) = 0$$

$$E_{uu}(0, 0) = \sum_{x, y} 2w(x, y) I_x(x, y) I_x(x, y)$$

$$E_u(0, 0) = 0$$

$$E_{vv}(0, 0) = \sum_{x, y} 2w(x, y) I_y(x, y) I_y(x, y)$$

$$E_v(0, 0) = 0$$

$$E_{uv}(0, 0) = \sum_{x, y} 2w(x, y) I_x(x, y) I_y(x, y)$$

# Corner Detection: Mathematics

The quadratic approximation simplifies to

$$E(u, v) \approx [u \ v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

where  $M$  is a *second moment matrix* computed from image derivatives:

$$M = \sum_{x,y} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Each product is  
a rank 1 2x2

Without  
weight

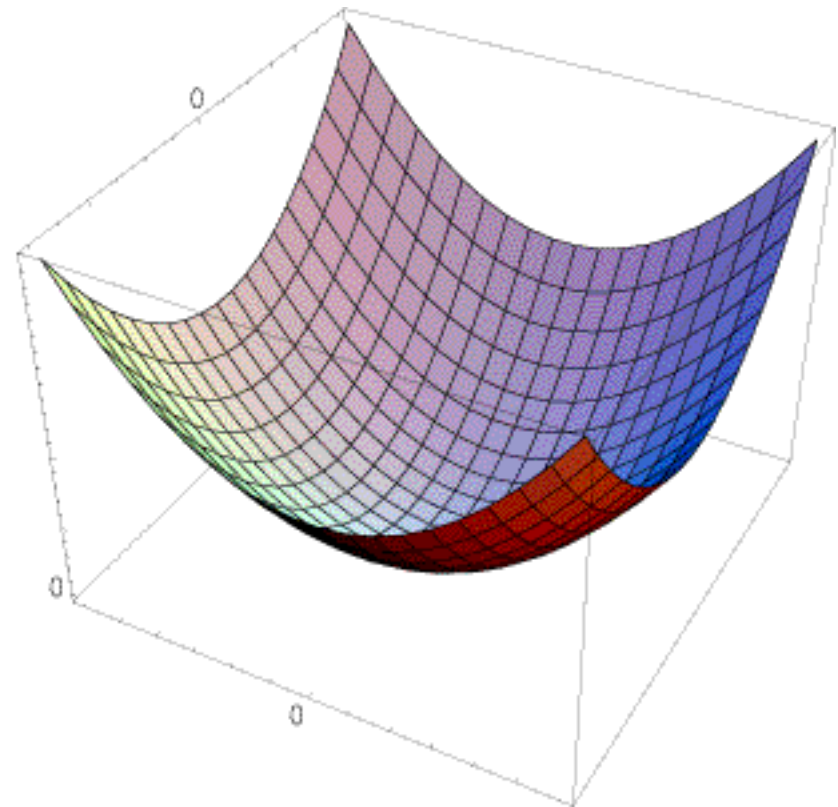
$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \ I_y] = \sum \nabla I (\nabla I)^T$$

# Interpreting the second moment matrix

The surface  $E(u, v)$  is locally approximated by a quadratic form.

$$E(u, v) \approx [u \ v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum_{x,y} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



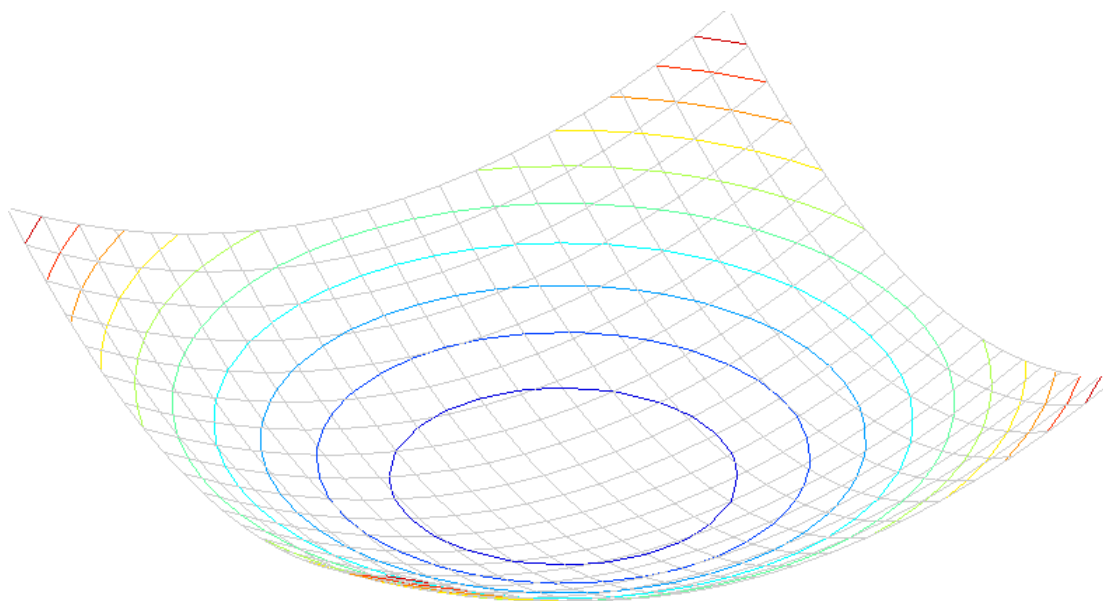


# Interpreting the second moment matrix

Consider a constant “slice” of  $E(u, v)$ :  $[u \ v] M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$

$$I_x^2 u^2 + 2I_x I_y uv + I_y^2 v^2 = k$$

This is the equation of an ellipse.



# Interpreting the second moment matrix

First, consider the axis-aligned case  
where gradients are either horizontal or  
vertical

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

If either  $\lambda$  is close to 0, then this is **not** a corner, so  
look for locations where both are large.

# Interpreting the second moment matrix

First, consider the axis-aligned case  
where gradients are either horizontal or  
vertical

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

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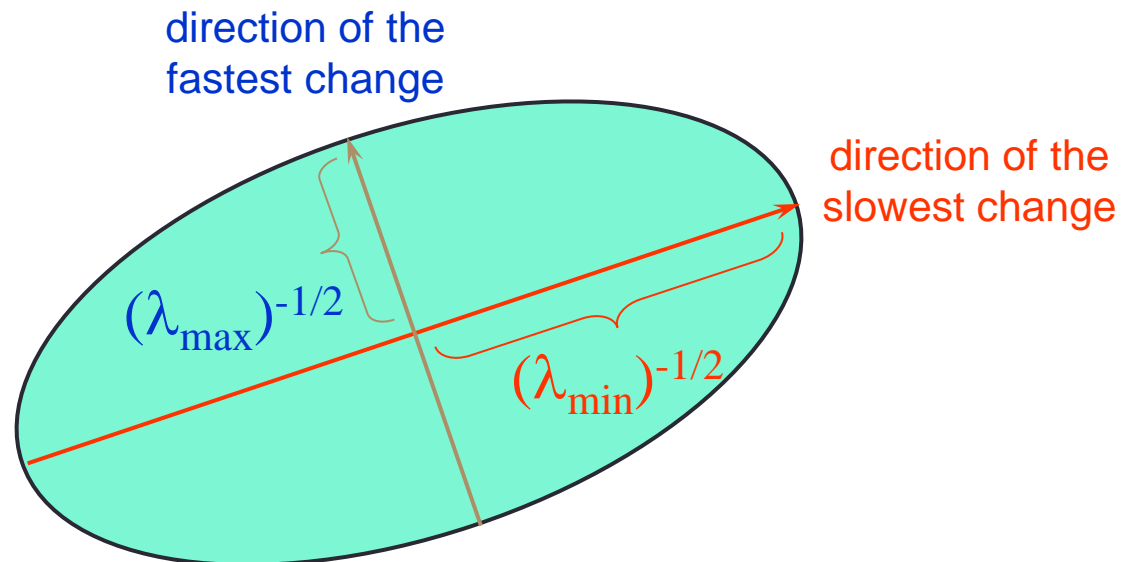
# Interpreting the second moment matrix

Consider a horizontal “slice” of  $E(u, v)$ :  $[u \ v] M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$

This is the equation of an ellipse.

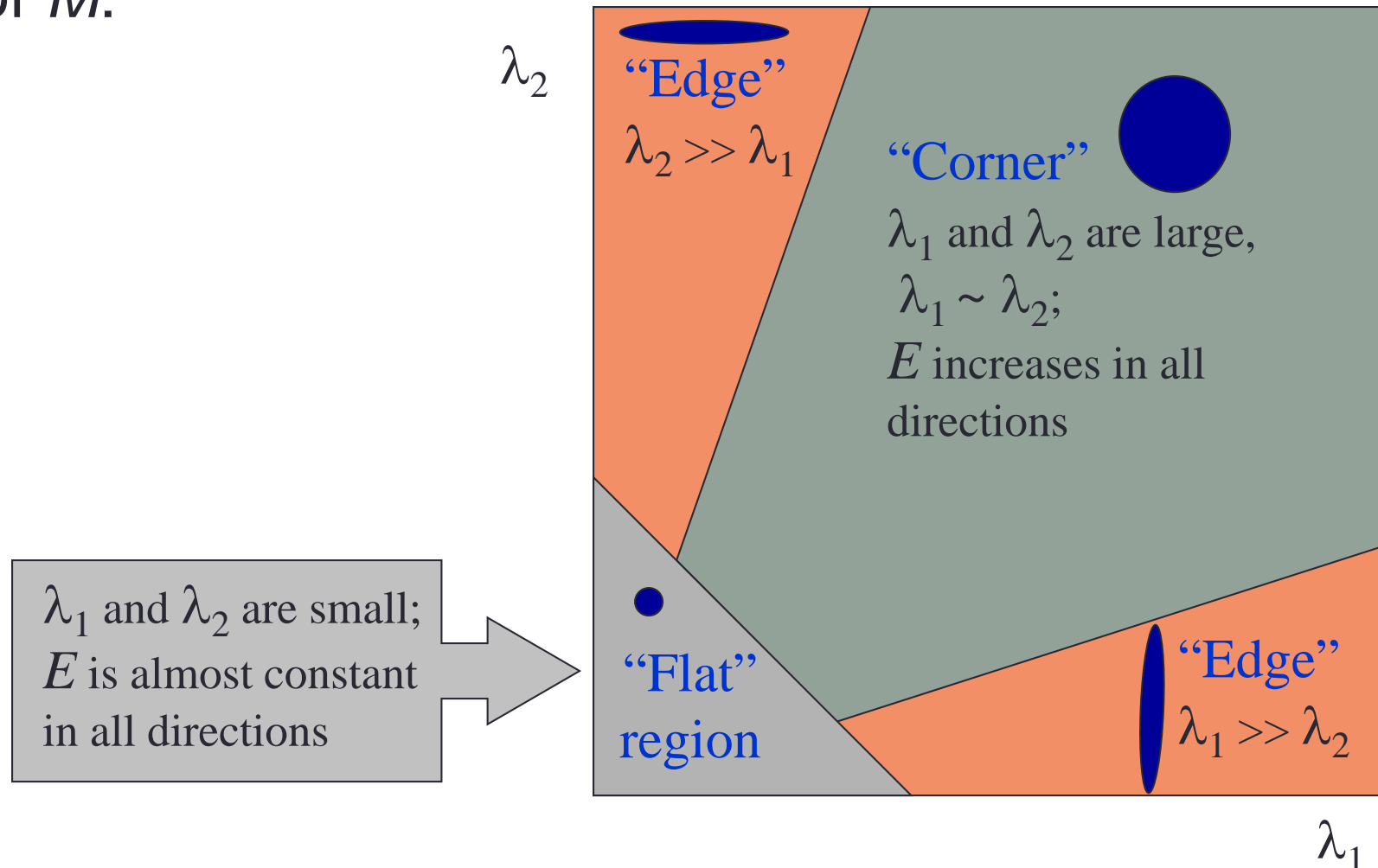
Diagonalization of  $M$ : 
$$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by  $R$



# Interpreting the eigenvalues

Classification of image points using eigenvalues of  $M$ :

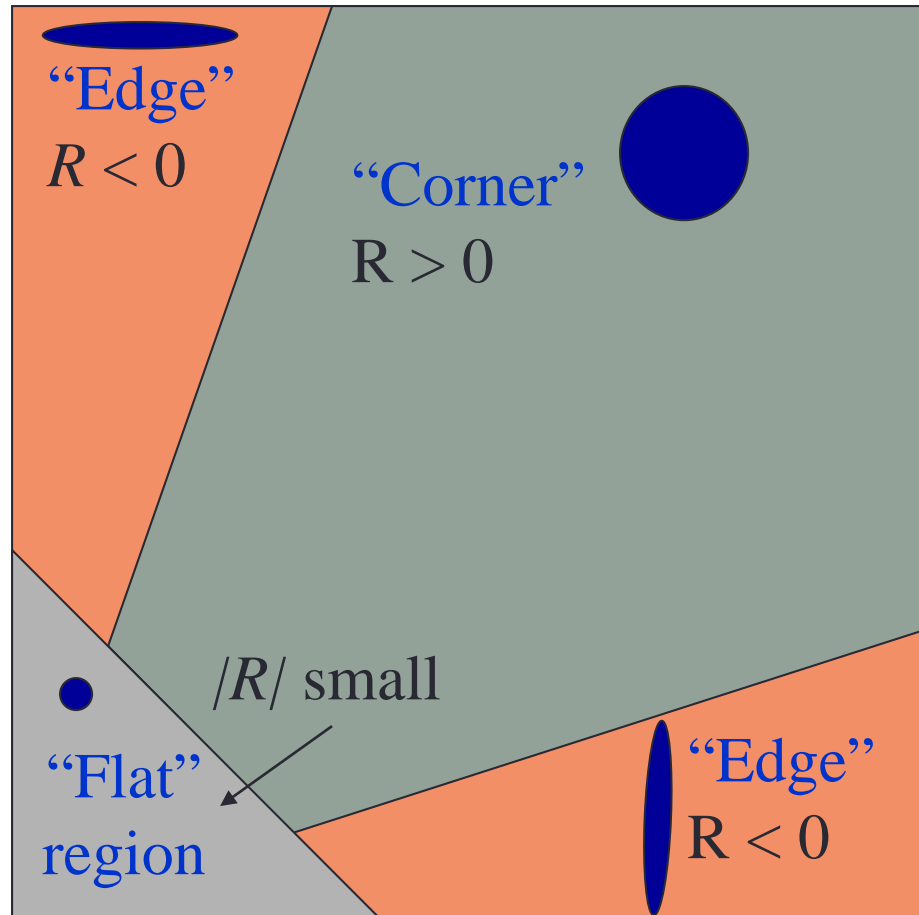


# Harris corner response function

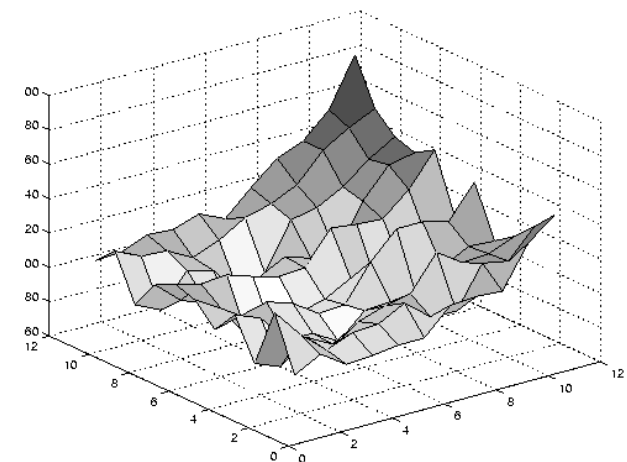
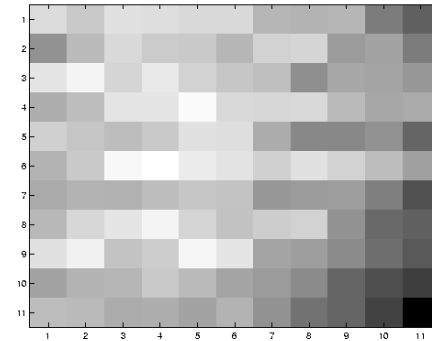
$$R = \det(M) - \alpha \text{trace}(M)^2 = \lambda_1 \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2$$

$\alpha$ : constant (0.04 to 0.06)

- $R$  depends only on eigenvalues of  $M$ , but don't compute them (no sqrt, so really fast!)
- $R$  is large for a **corner**
- $R$  is negative with large magnitude for an **edge**
- $|R|$  is small for a **flat** region



# Low texture region

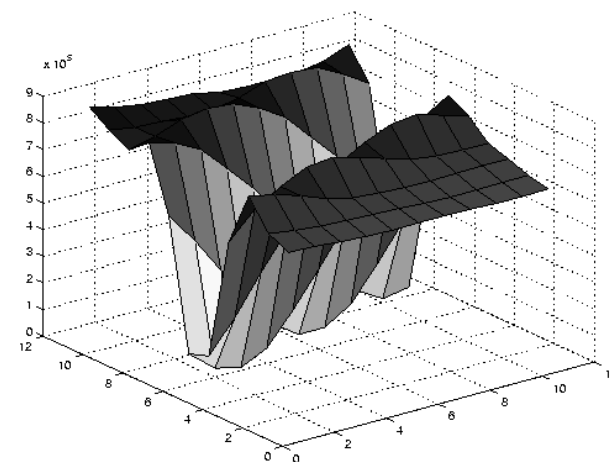
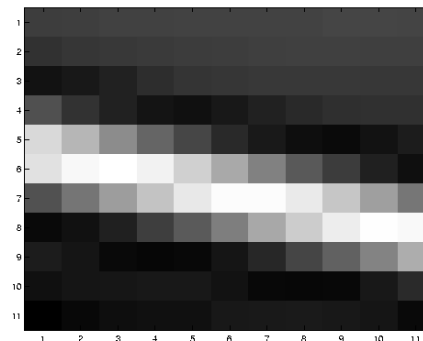


$$\sum \nabla I (\nabla I)^T$$

- gradients have small magnitude
- small  $\lambda_1$ , small  $\lambda_2$



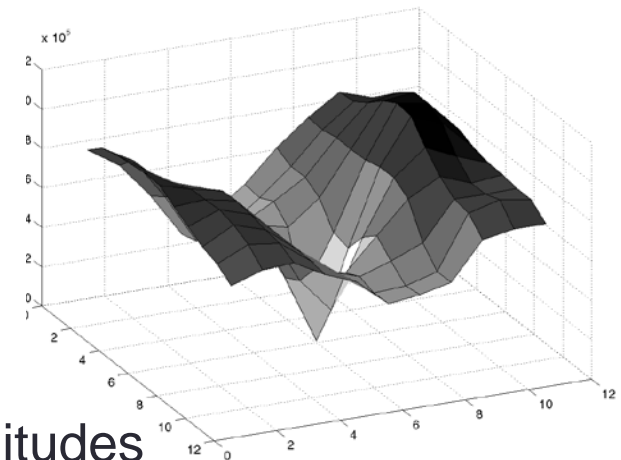
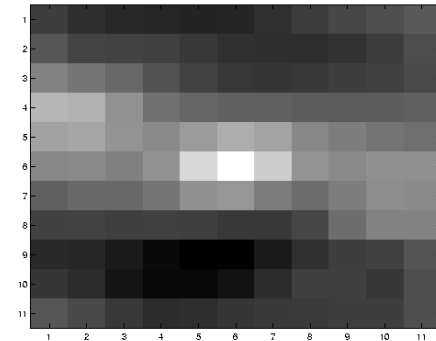
# Edge



$$\sum \nabla I (\nabla I)^T$$

- large gradients, all the same
- large  $\lambda_1$ , small  $\lambda_2$

# High textured region



$$\sum \nabla I (\nabla I)^T$$

- gradients are different, large magnitudes
- large  $\lambda_1$ , large  $\lambda_2$

# Harris detector: Algorithm

1. Compute Gaussian derivatives at each pixel
2. Compute second moment matrix  $M$  in a Gaussian window around each pixel
3. Compute corner response function  $R$
4. Threshold  $R$
5. Find local maxima of response function (nonmaximum suppression)

C.Harris and M.Stephens. ["A Combined Corner and Edge Detector."](#)  
*Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

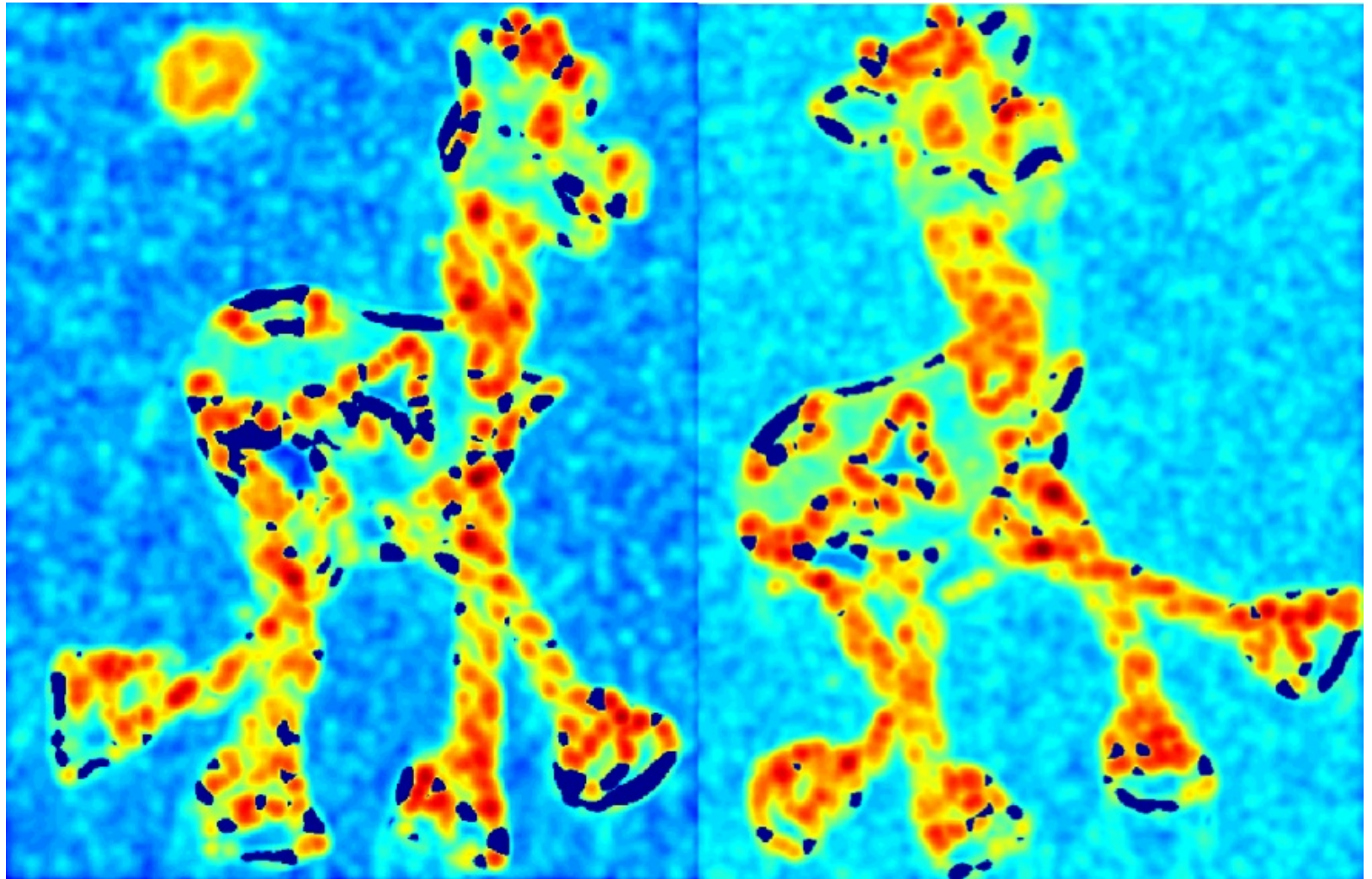
# Harris Detector: Workflow





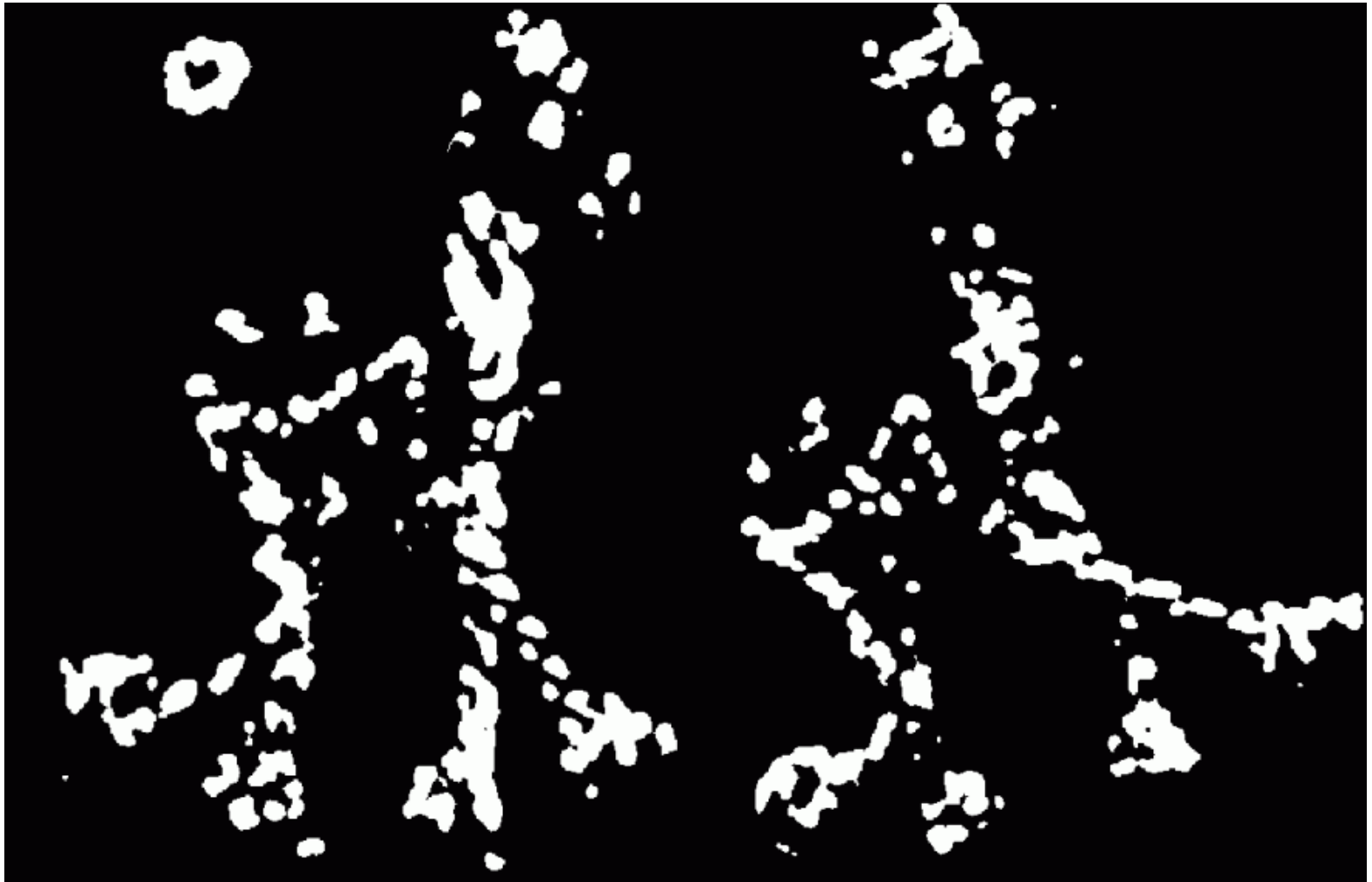
# Harris Detector: Workflow

Compute corner response  $R$



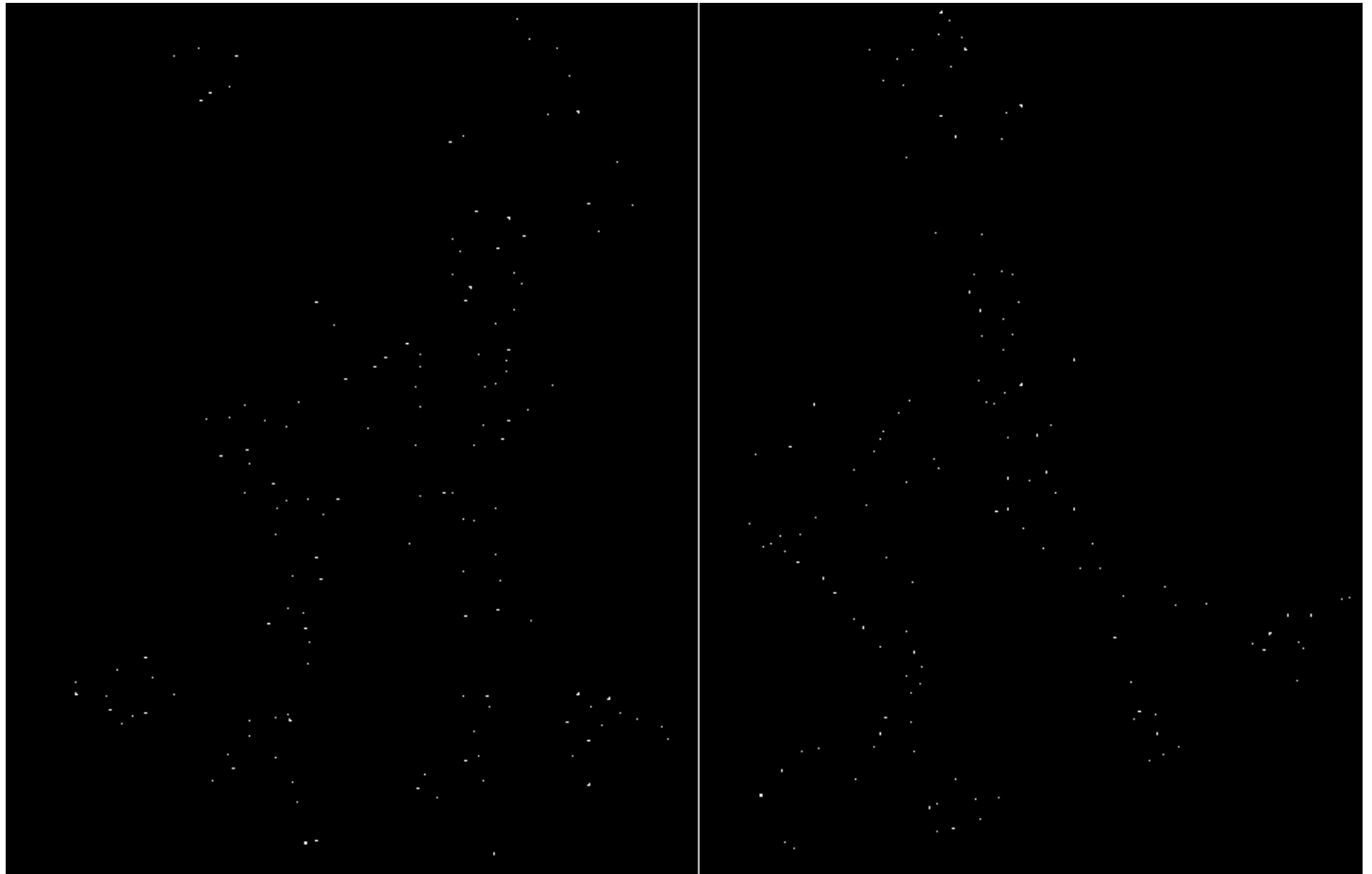
# Harris Detector: Workflow

Find points with large corner response:  $R > \text{threshold}$



# Harris Detector: Workflow

Take only the points of local maxima of  $R$



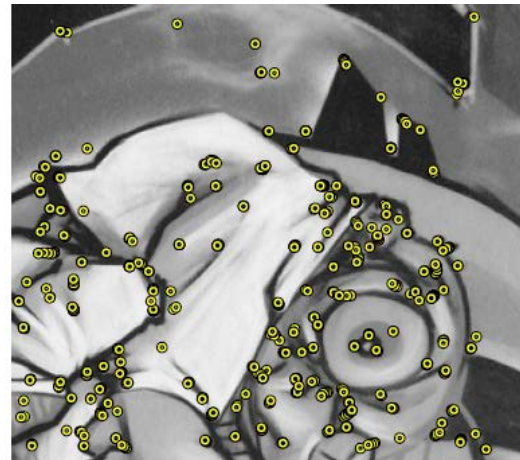
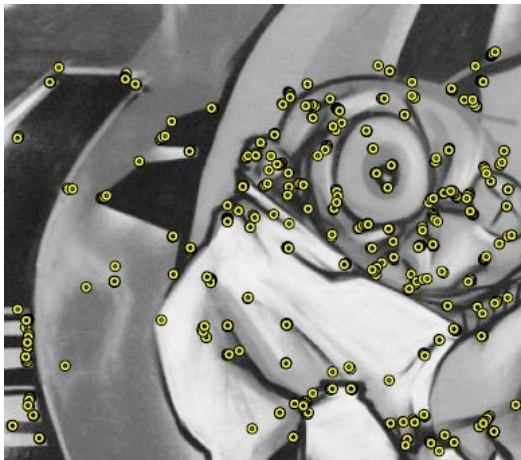


# Harris Detector: Workflow



# Other corners:

- Shi-Tomasi '94:
  - “Cornersness” =  $\min(\lambda_1, \lambda_2)$  Find local maximums
  - `cvGoodFeaturesToTrack(...)`
  - Reportedly better for region undergoing affine deformations

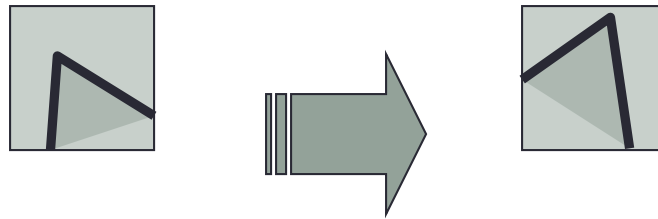


- Brown, M., Szeliski, R., and Winder, S. (2005):  $\frac{\det M}{\text{tr } M} = \frac{\lambda_0 \lambda_1}{\lambda_0 + \lambda_1}$
- there are others...

# Harris Detector: Some Properties

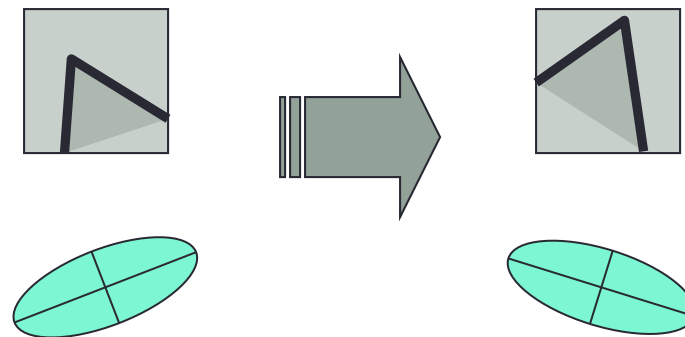
# Harris Detector: Some Properties

- Rotation invariance?



# Harris Detector: Some Properties

- Rotation invariance



Ellipse rotates but its shape (i.e. eigenvalues) remains the same

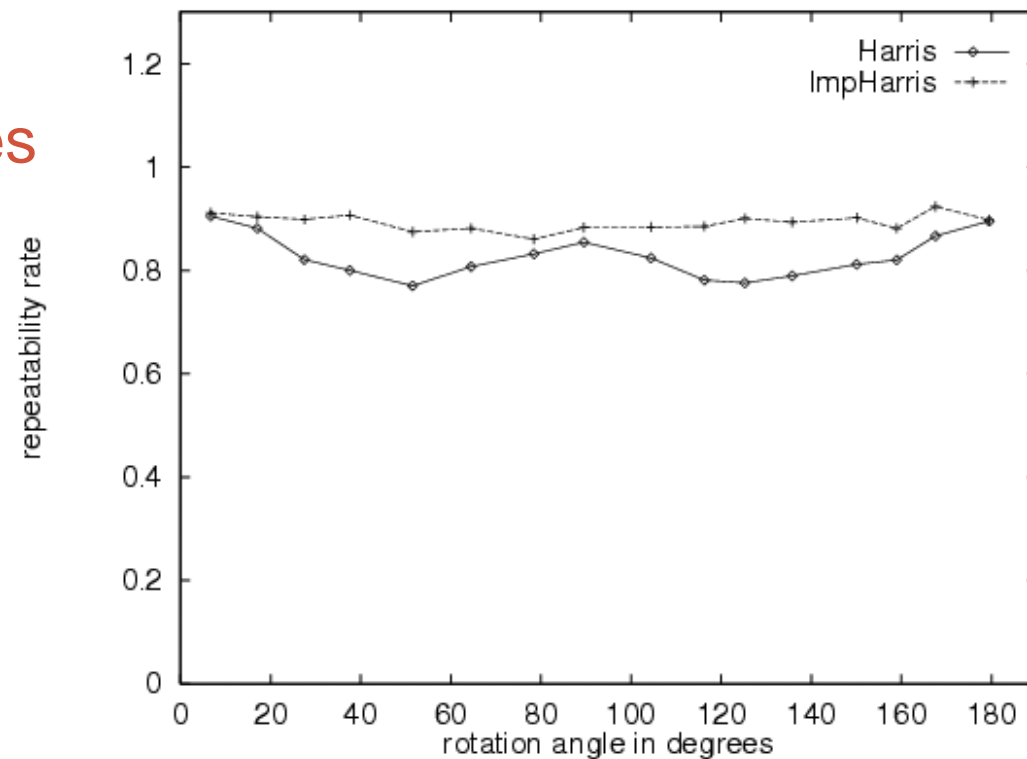
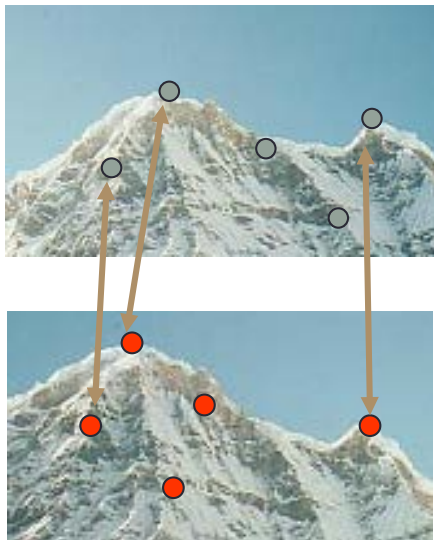
*Corner response  $R$  is invariant to image rotation*

# Rotation Invariant Detection

## Harris Corner Detector

### Repeatability rate:

$$\frac{\# \text{ correspondences}}{\# \text{ possible correspondences}}$$



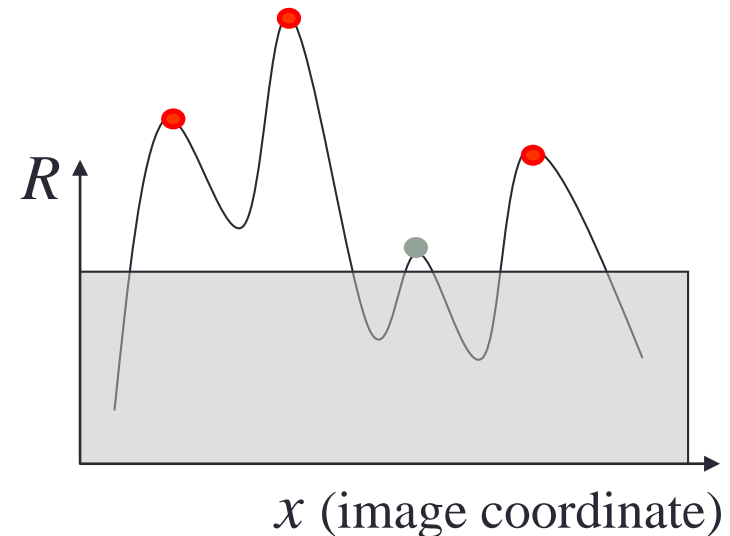
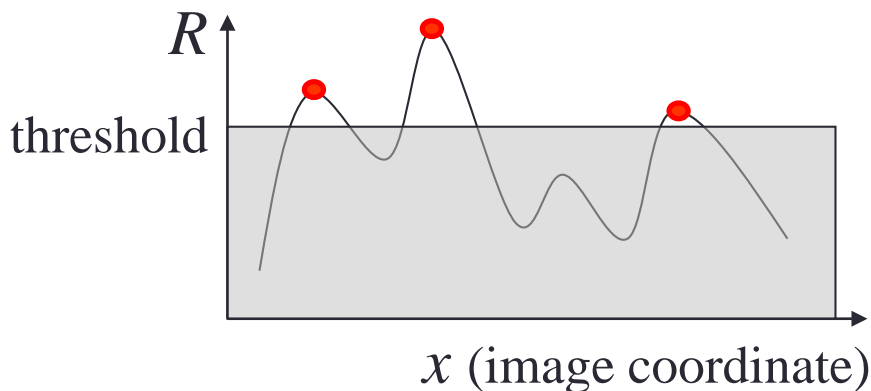
# Harris Detector: Some Properties

- Invariance to image intensity change?



# Harris Detector: Some Properties

- Partial invariance to additive and multiplicative intensity changes (threshold issue for multiplicative)
  - ✓ Only derivatives are used  $\Rightarrow$  invariance to intensity shift  $I \rightarrow I + b$
  - ✓ Intensity scale:  $I \rightarrow a I$

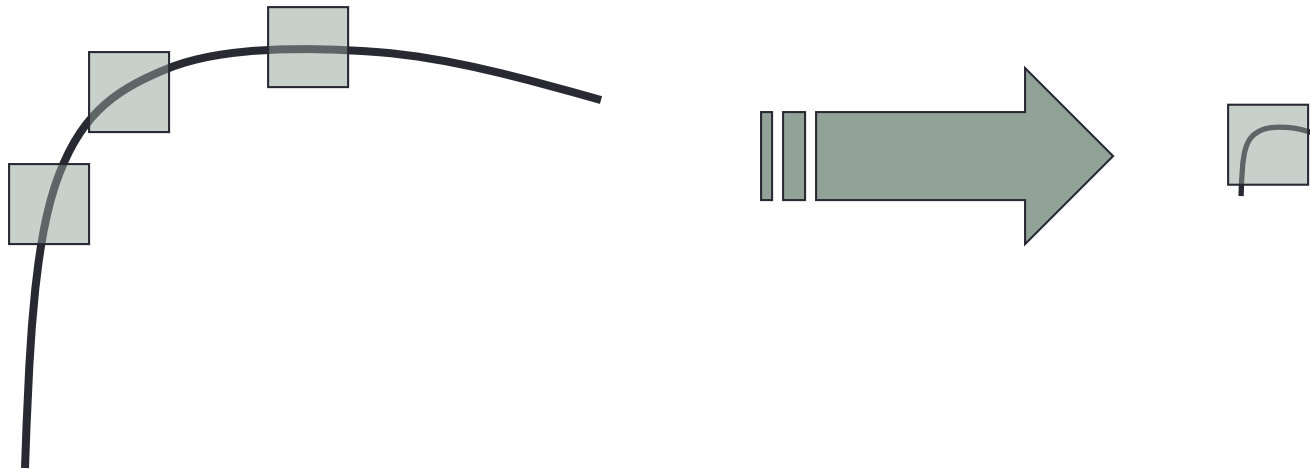


# Harris Detector: Some Properties

- Invariant to image scale?

# Harris Detector: Some Properties

- **Not invariant to *image scale*!**

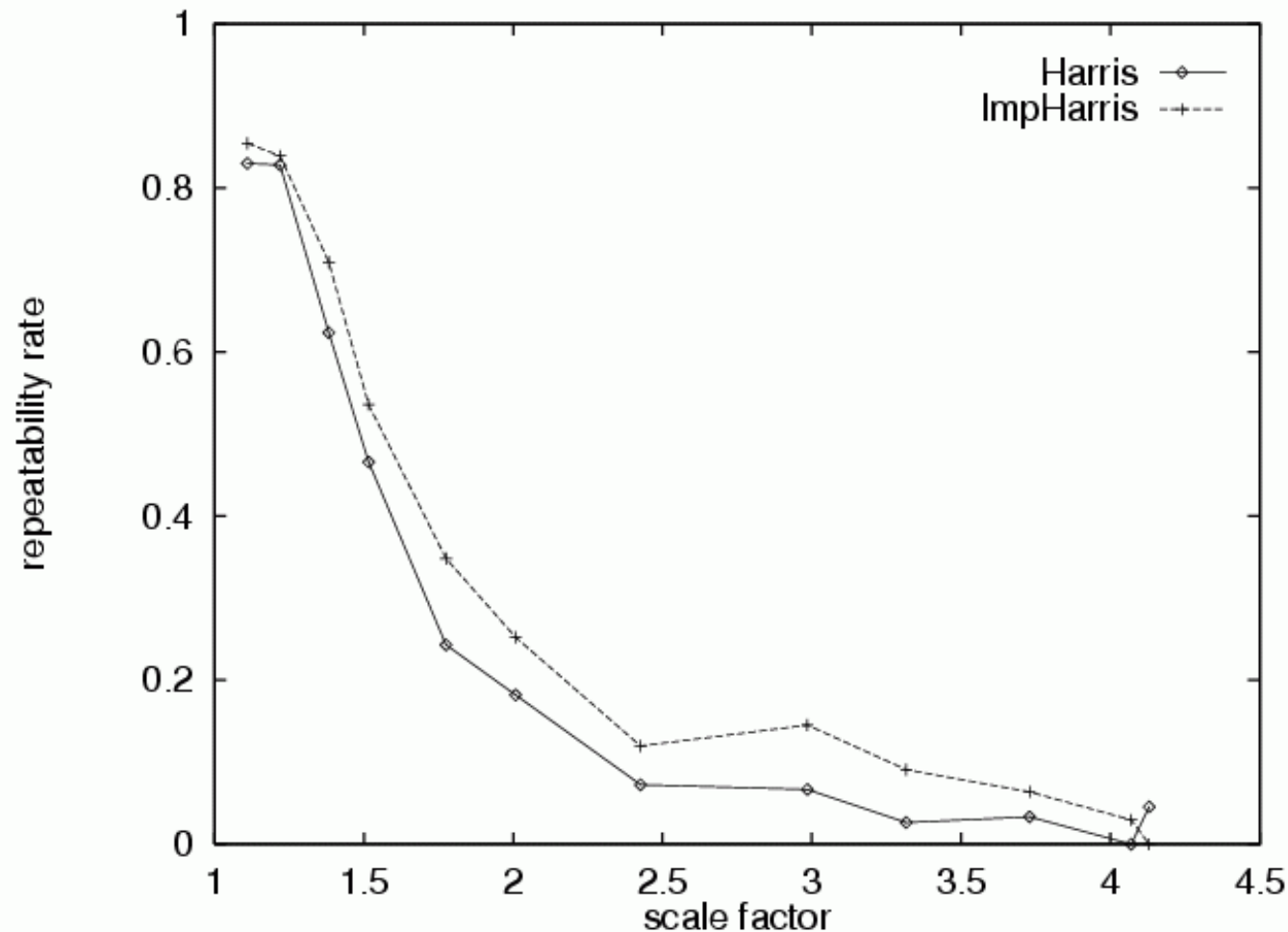


All points will be  
classified as **edges**

**Corner !**

# Harris Detector: Some Properties

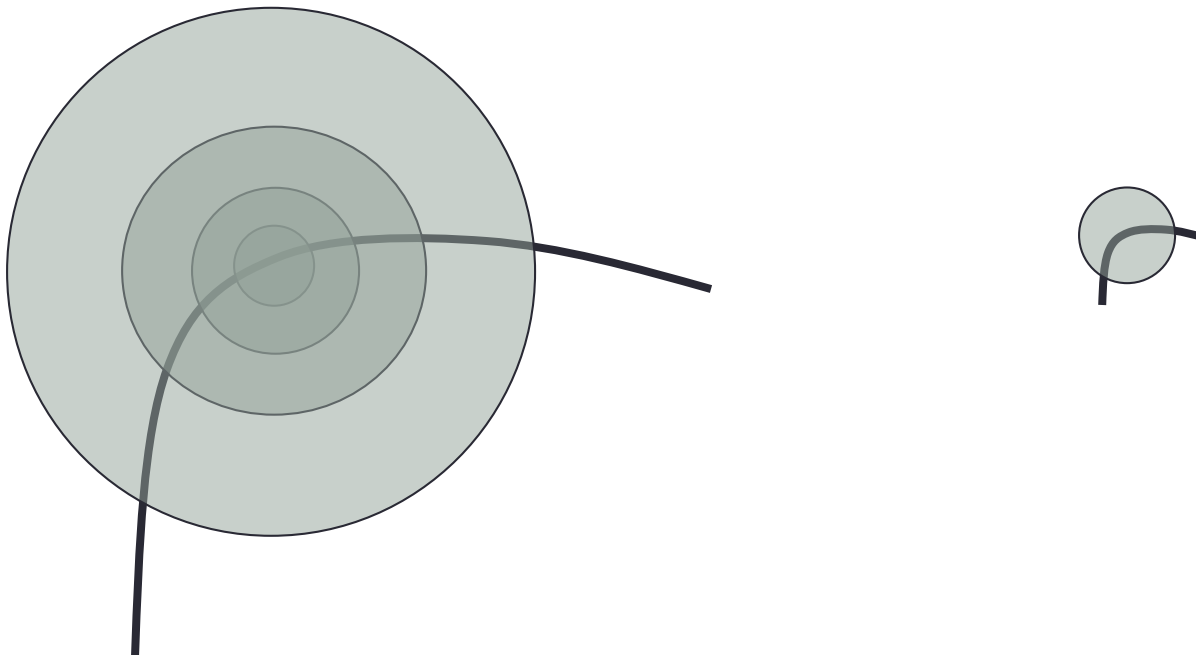
- Quality of Harris detector for different scale changes



***\*IF\*** we want scale invariance...*

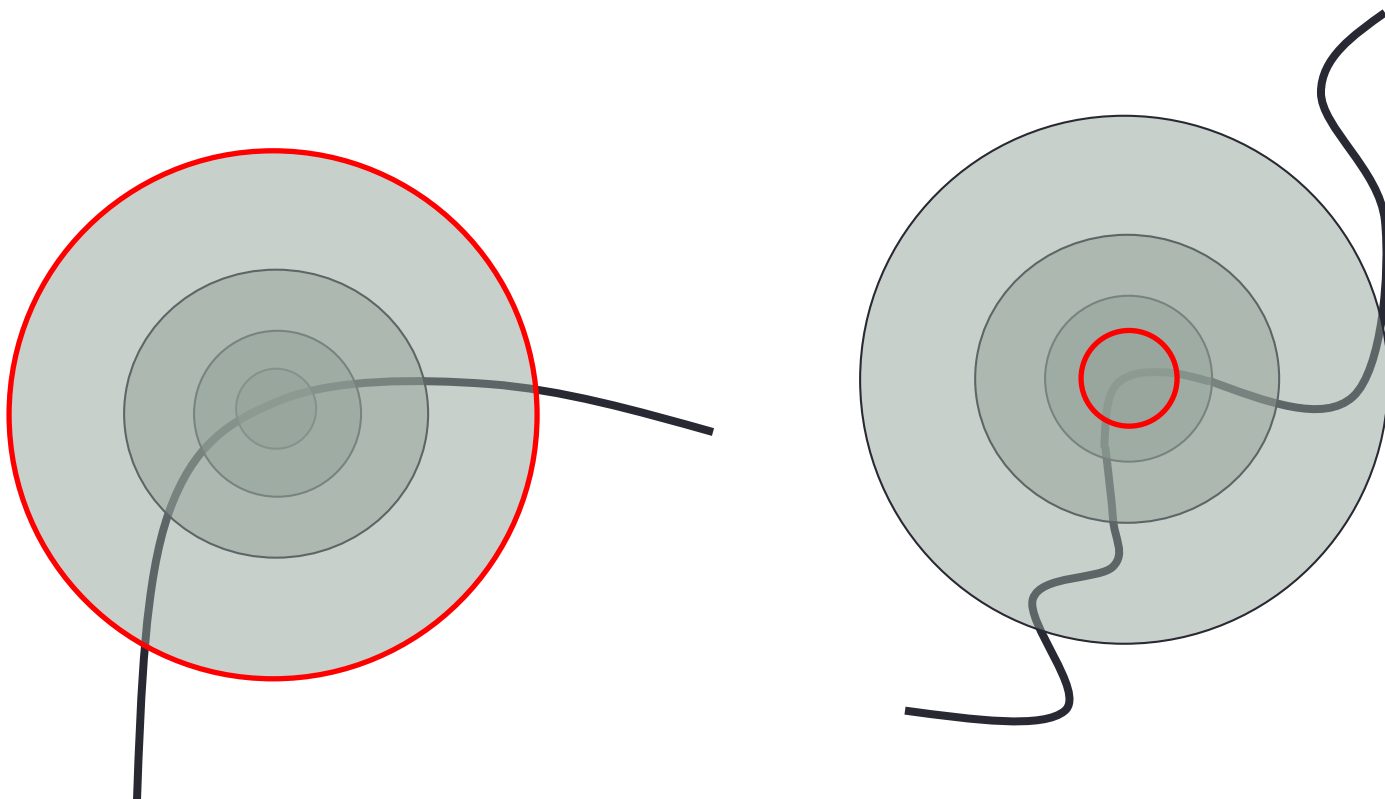
# Scale Invariant Detection

- Consider regions (e.g. circles) of different sizes around a point
- Regions of corresponding sizes will look the same in both images



# Scale Invariant Detection

- The problem: how do we choose corresponding circles independently in each image?



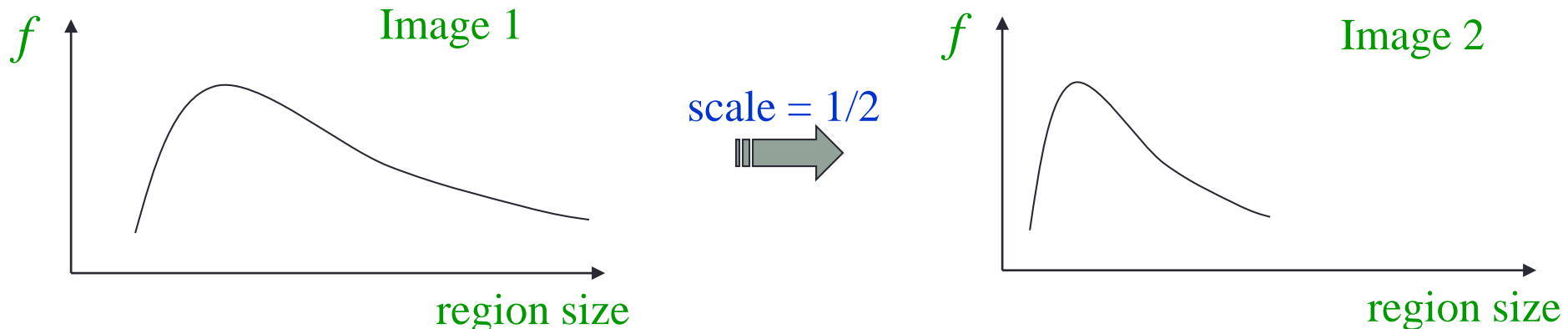
# Scale Invariant Detection

- Solution:

- Design a function on the region (circle), which is “scale invariant” (the same for corresponding regions, even if they are at different scales)

Example: average intensity. For corresponding regions (even of different sizes) it will be the same.

For some given point in one image, we can consider it as a function of region size (circle radius)





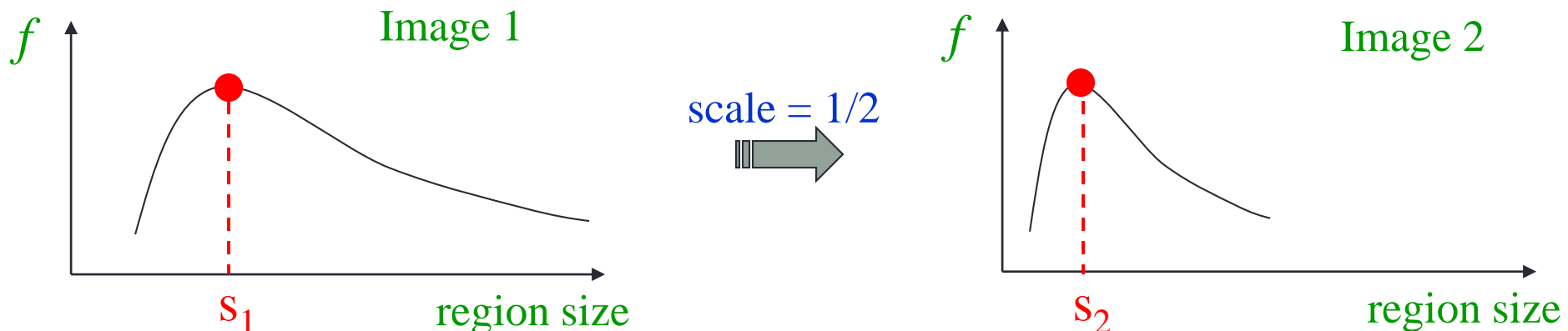
# Scale Invariant Detection

- Common approach:

Take a local maximum of this function

Observation: region size, for which the maximum is achieved, should be *invariant* to image scale.

Important: this scale invariant region size is found in each image **independently!**



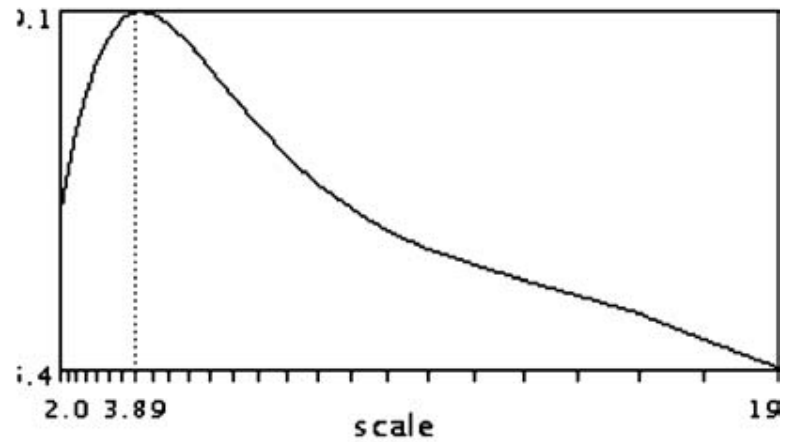
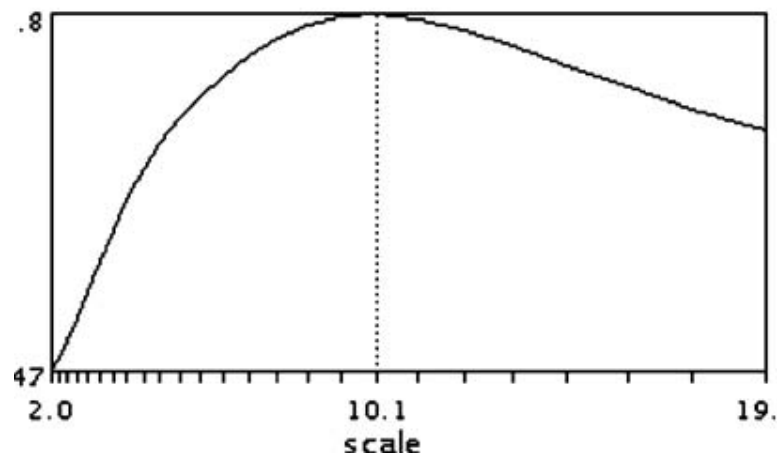
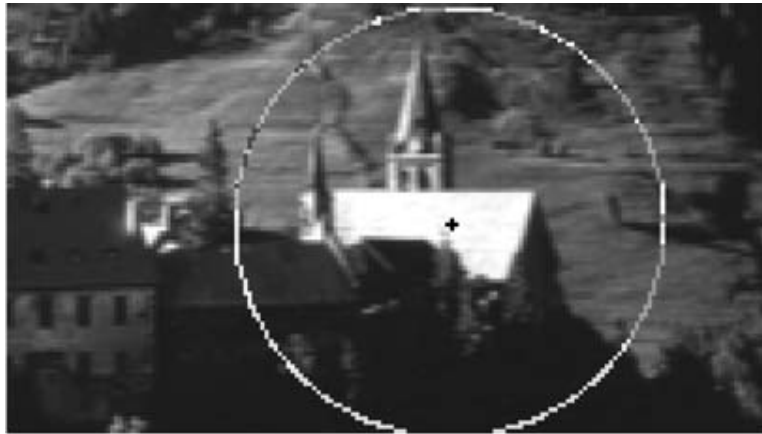
# Scale Invariant Detection

- A “good” function for scale detection:  
has one stable sharp peak



- For usual images: a good function would be a one which responds to contrast (sharp local intensity change)

# Scale sensitive response



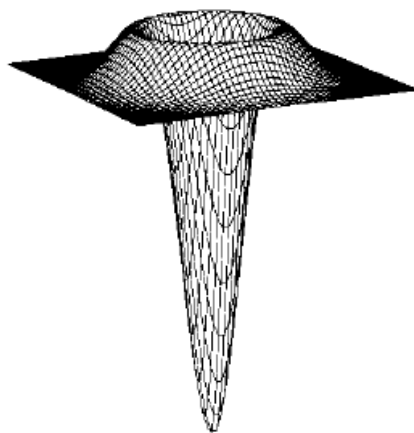
# Scale Invariant Detection

Function is just application of a kernel:  $f = \text{Kernel} * \text{Image}$

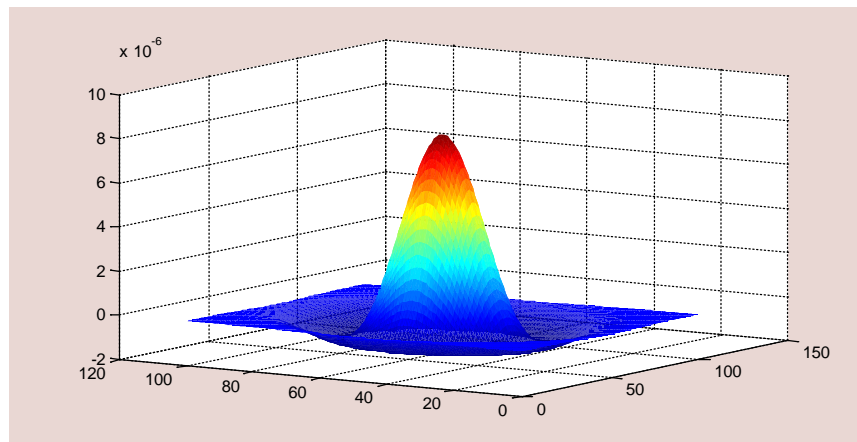
(Laplacian of Gaussian - LoG)

$$L = \sigma^2 \left( G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

Laplacian of Gaussian



$$\nabla^2 h_\sigma(u, v)$$



# Scale Invariant Detection

- Functions for determining scale

$$f = \text{Kernel} * \text{Image}$$

Kernels:

$$L = \sigma^2 \left( G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

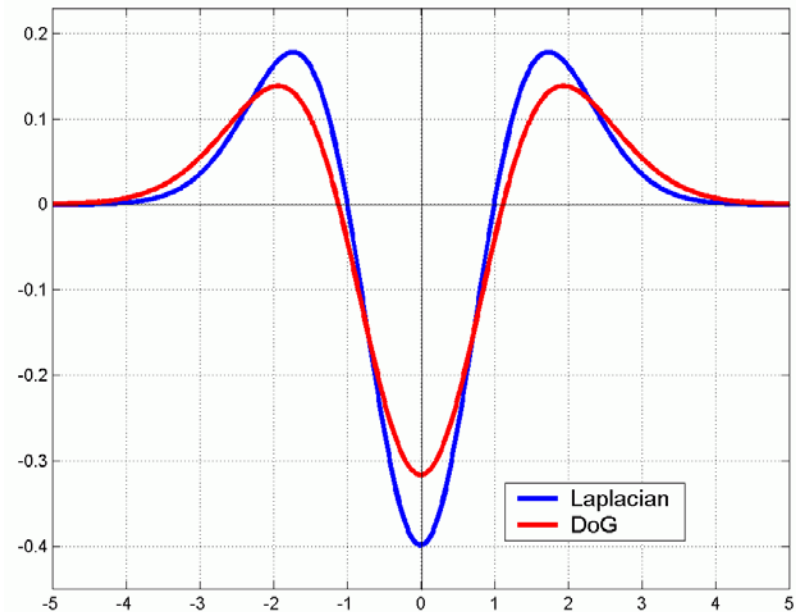
(Laplacian)

$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)

where Gaussian

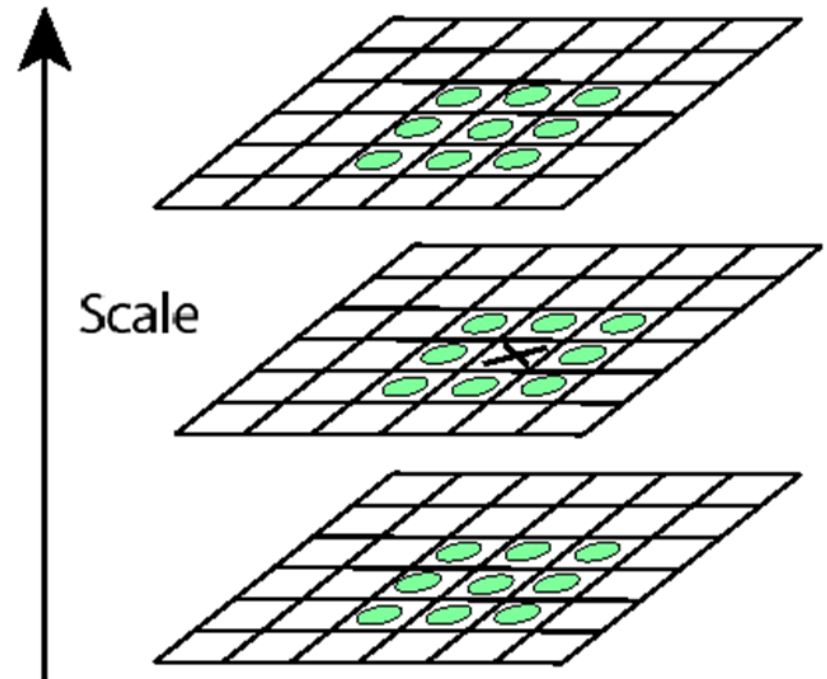
$$G(x, y, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2 + y^2}{2\sigma^2}}$$



Note: both kernels are invariant to  
*scale* and *rotation*

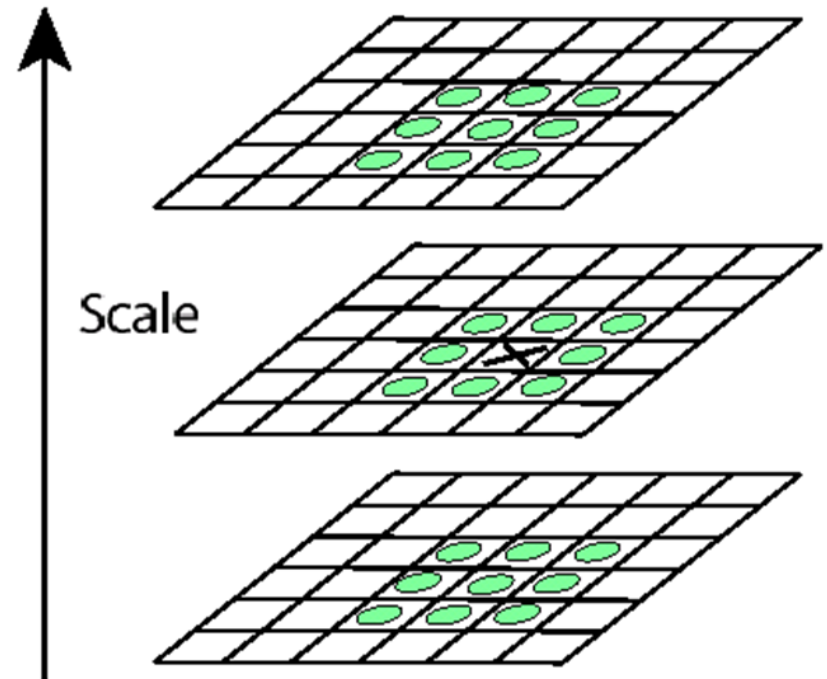
# Key point localization

- General idea: find robust extremum (maximum or minimum) both in space and in scale.



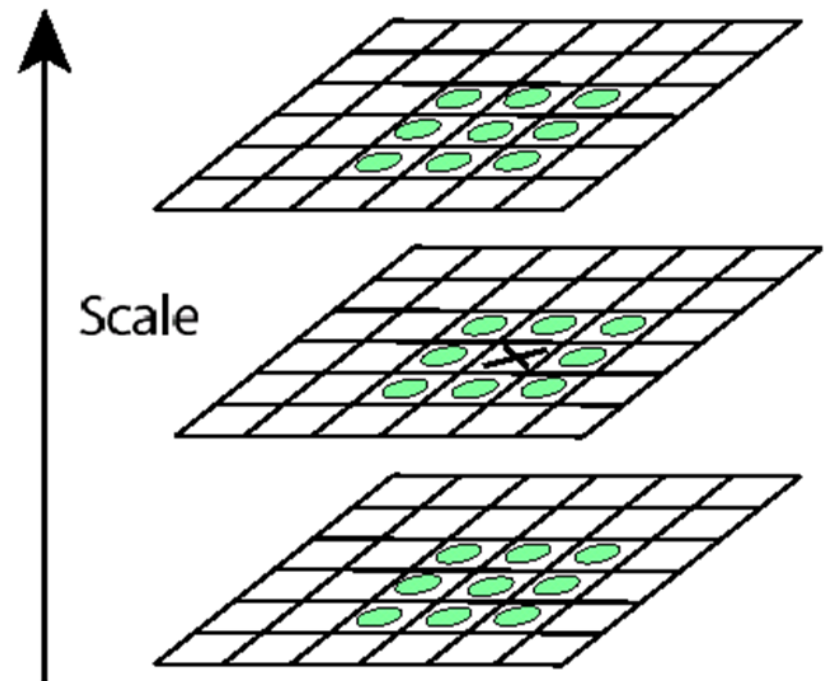
# Key point localization

- SIFT: **S**cale **I**nvariant **F**eature **T**ransform
- Specific suggestion: use DoG pyramid to find maximum values (remember edge detection?) – then eliminate “edges” and pick only corners.



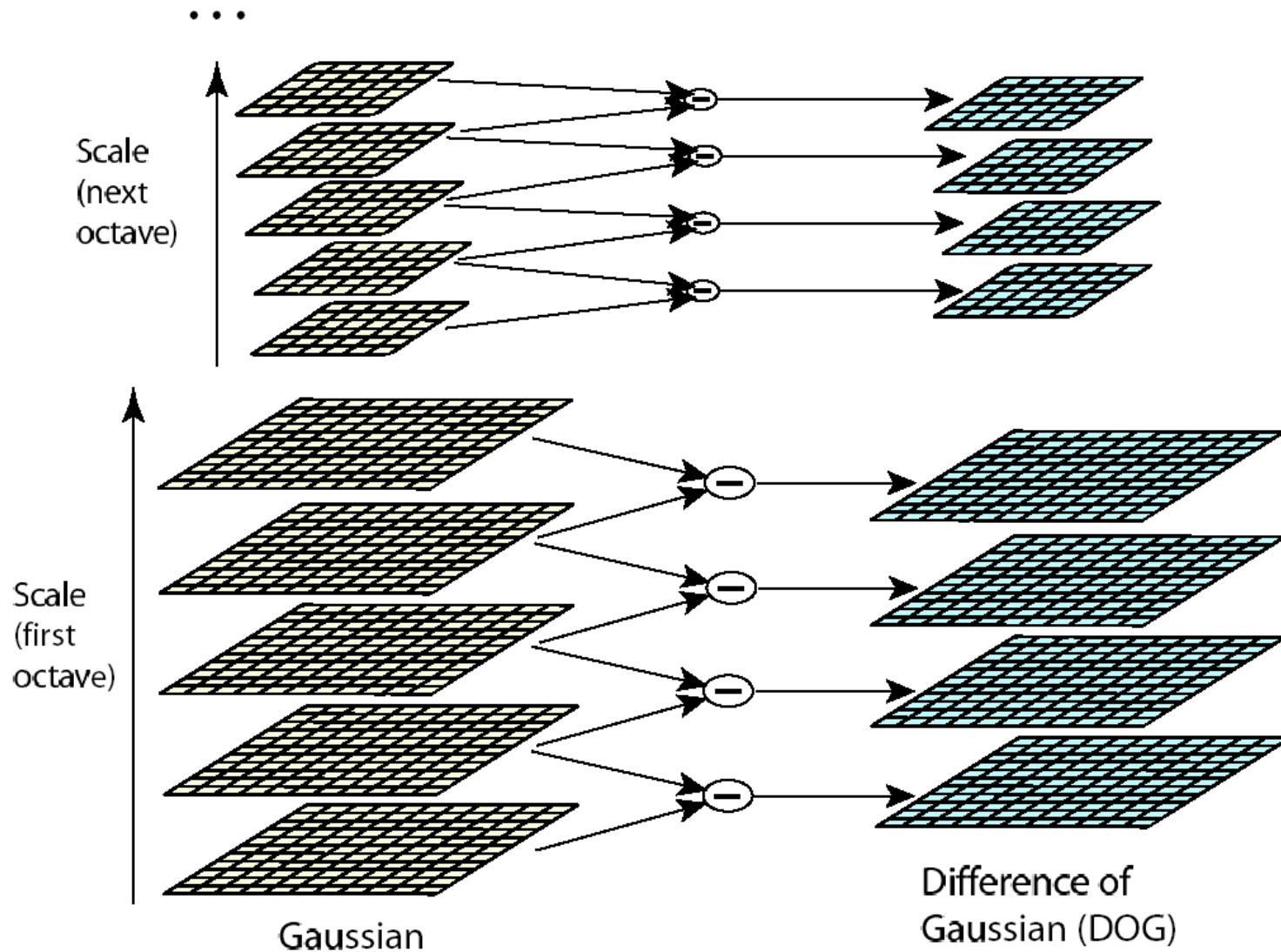
# Key point localization

*(Each point is compared to its 8 neighbors in the current image and 9 neighbors each in the scales above and below.)*





# Scale space processed one octave at a time



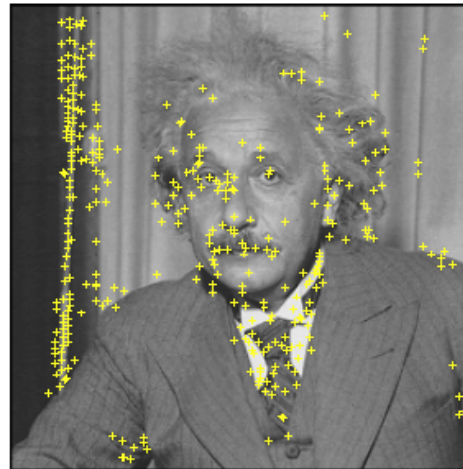
# Extrema at different scales



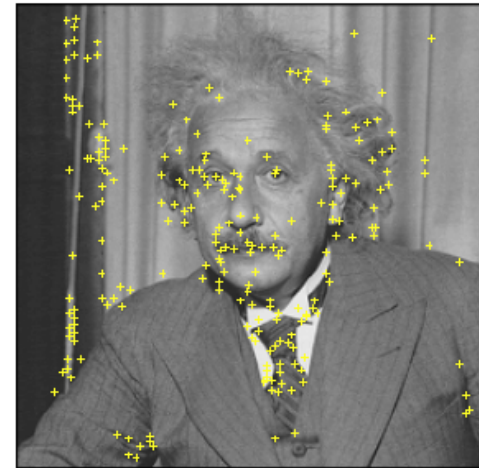
# Remove low contrast, edge bound



Extrema points



Contrast  $> C$



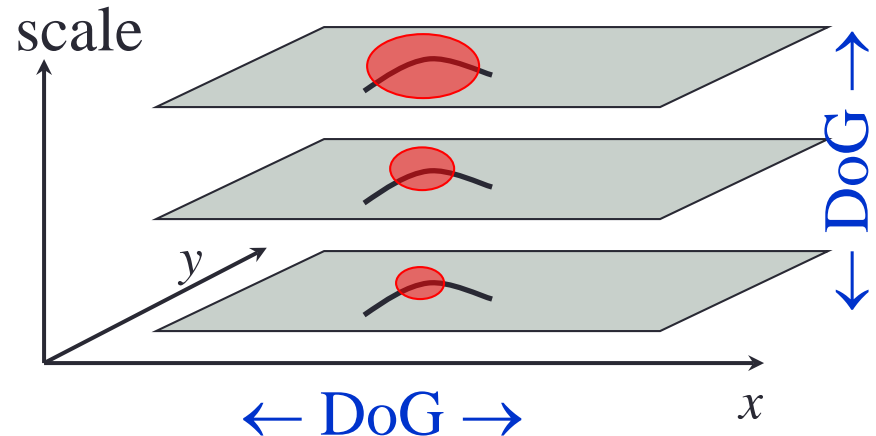
Not on edge

# Scale Invariant Detectors

- **SIFT (Lowe)<sup>1</sup>**

*Find local maximum of:*

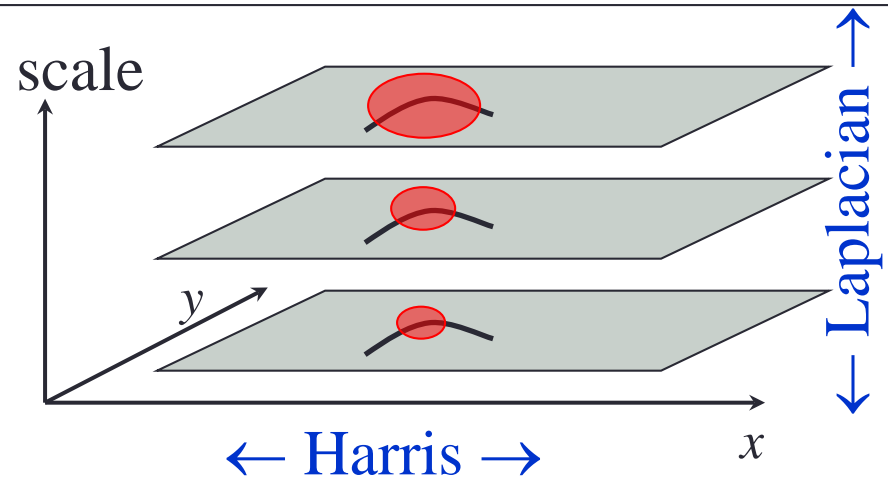
- Difference of Gaussians in space and scale



- **Harris-Laplacian<sup>2</sup>**

*Find local maximum of:*

- Harris corner detector in space (image coordinates)
- Laplacian in scale



<sup>1</sup>D.Lowe. “Distinctive Image Features from Scale-Invariant Keypoints”. IJCV 2004

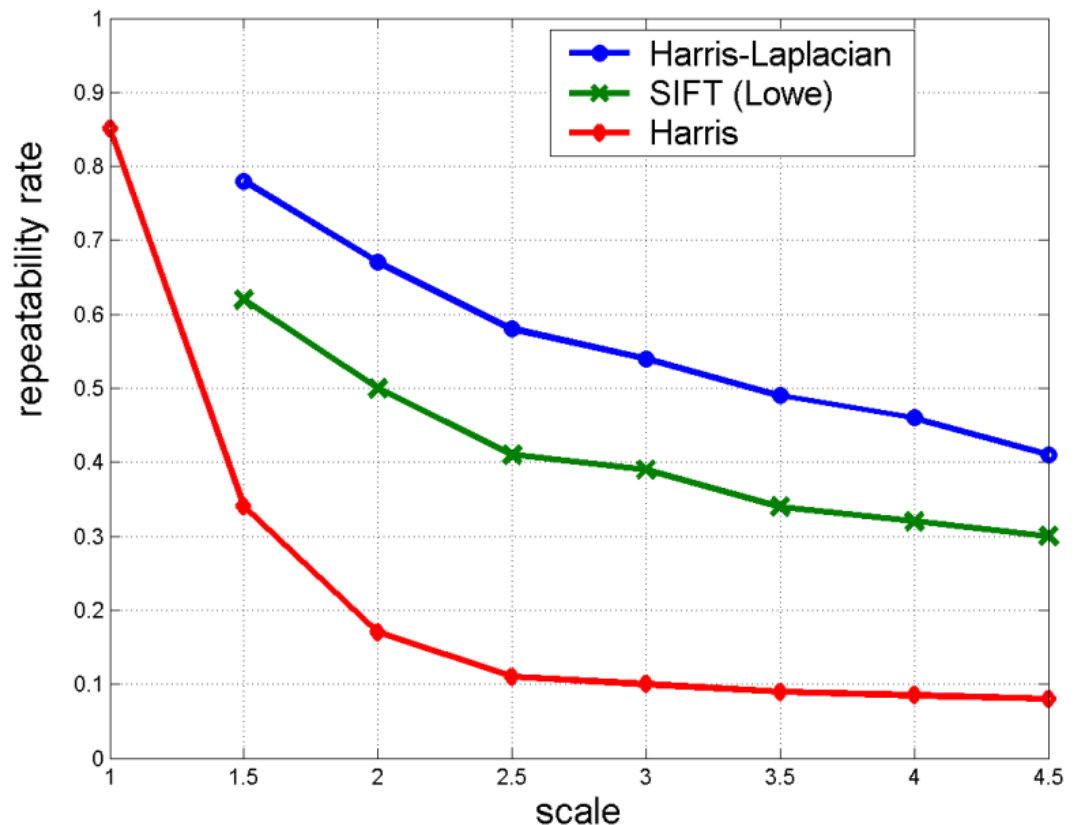
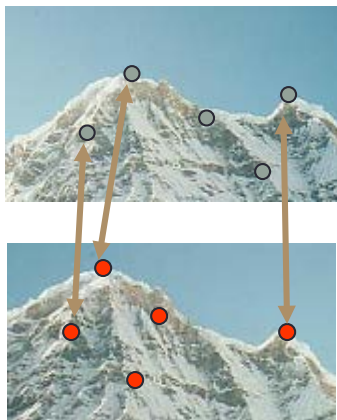
<sup>2</sup>K.Mikolajczyk, C.Schmid. “Indexing Based on Scale Invariant Interest Points”. ICCV 2001

# Scale Invariant Detectors

- Experimental evaluation of detectors w.r.t. scale change

Repeatability rate:

$$\frac{\# \text{ correspondences}}{\# \text{ possible correspondences}}$$



# Scale Invariant Detection: Summary

- Given: two images of the same scene with a *large* scale difference between them
- Goal: find the same interest points independently in each image
- Solution: search for maxima of suitable functions in scale and in space (over the image)

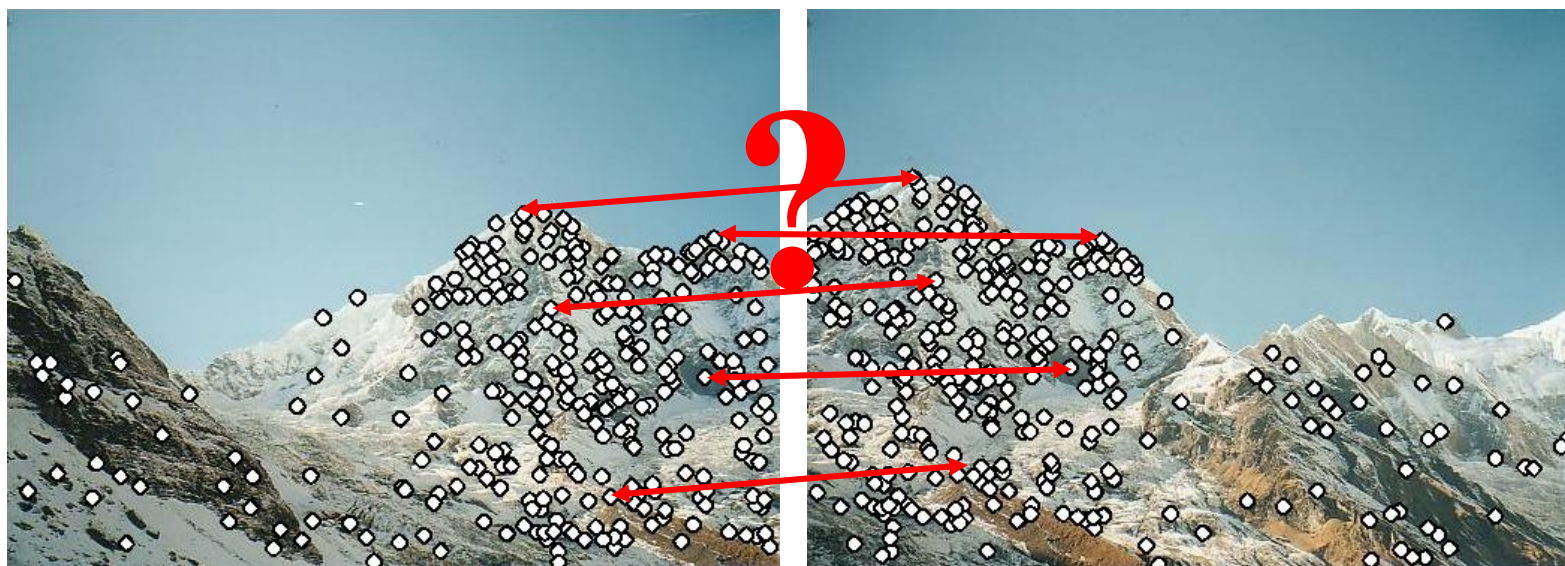
## Methods:

1. **SIFT** [Lowe]: maximize Difference of Gaussians over scale and space
2. **Harris-Laplacian** [Mikolajczyk, Schmid]: maximize Laplacian over scale, Harris' measure of corner response over the image



# Point Descriptors

- We know how to detect points
- Next question: How to match them?



Point descriptor should be:

1. Invariant
2. Distinctive

# Next time...

- SIFT, SURF, SFOP, oh my...